Strategic games as cybernetic systems

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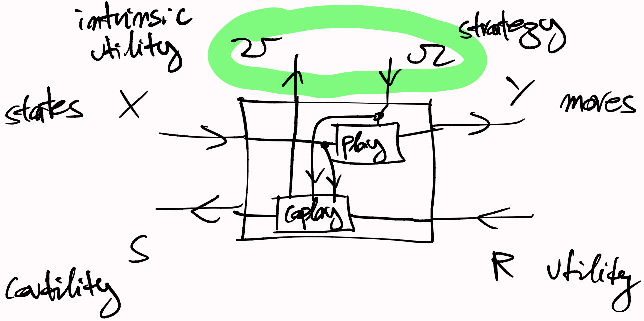
Outline

1. The issue with open games
2. Learning from learners
3. Feedback structures
4. A behavioural approach to Nash equilibria

ARENA



$$\mathrm{play} : \Omega \times X \longrightarrow Y$$

$$\mathrm{coplay} : \Omega \times X \times R \longrightarrow S \times \mho$$

selection function

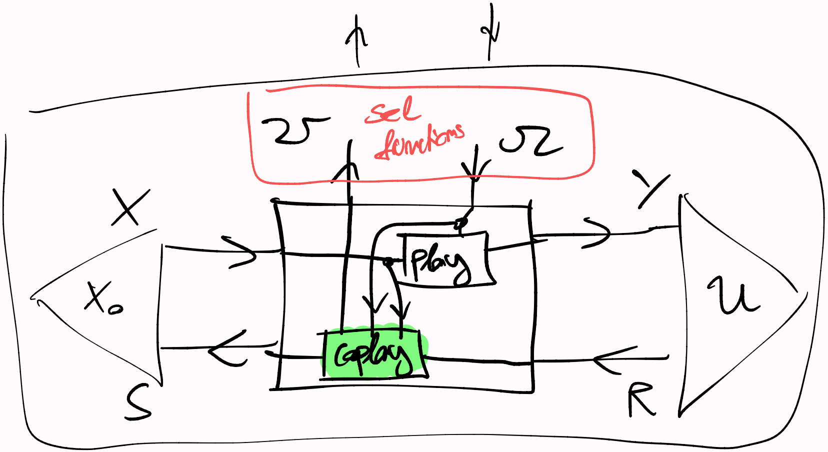
(eq predicate)

$$\varepsilon : (\Omega \to \mho) \to P\Omega$$

$$\|$$

$$\mathrm{argmax}$$

$$\mho = \mathbb{R}$$



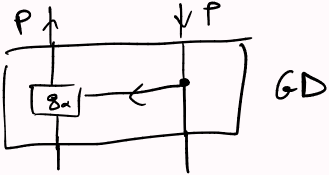
$\omega \in \Omega$ $\quad \forall$ player $\quad \mathcal{U}(\underbrace{\omega'}_{\text{ω with a unilateral deviation}}) \leq \mathcal{U}(\omega)$

$\alpha \in \mathbb{R}^+$

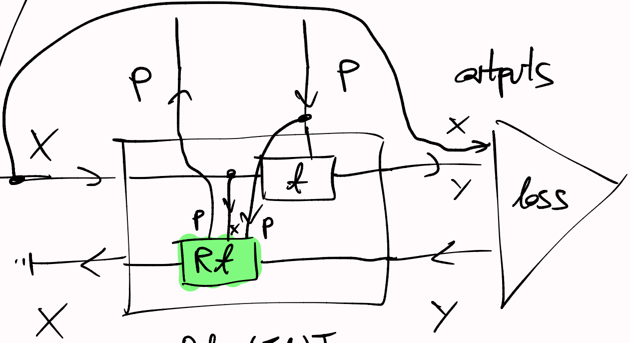
$g_\alpha : (p:P) \times (v:P) \longrightarrow P$

$(p,v) \longmapsto p - \alpha v$

P



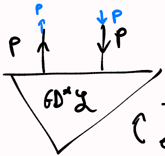
inputs



~ on X

$Rf = (Jf)^T$

"tangent" vectors on Y



$\cong P \longrightarrow P$

equilibria = fixpoints of $GD^\alpha \mathcal{L}$

$\ell : Y \longrightarrow \mathbb{R} \quad \in C^\infty(Y)$ <u>scalar field/function</u>

$\downarrow$

$$\begin{tikzcd} d\ell : TY \arrow[r, "lin"] \arrow[d] \arrow[dr, phantom, "\circlearrowright"] & \mathbb{R}\times\mathbb{R} \arrow[d] \\ \ell : Y \arrow[r] & \mathbb{R} \end{tikzcd}$$

$$\begin{tikzcd} TY \arrow[d, "\pi_Y"] \\ Y \end{tikzcd}$$

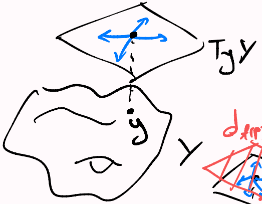
tangent ~~space~~ bundle

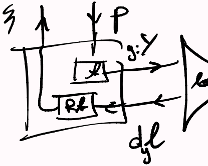
$y : Y \vdash T_yY$ type + vector space $(0, +, \cdot)$

$d\ell : (y:Y) \times \underbrace{T_yY \longrightarrow \mathbb{R}}_{\text{linear}}$

$\shortparallel$

functional over T_yY

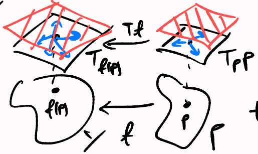




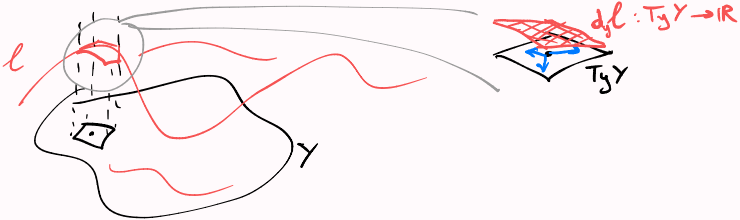
$T_pP \xrightarrow{f^*(d_{f(p)}\ell)} \mathbb{R}$

$T_pP \xrightarrow{Tf}$

$d_{f(p)}\ell : T_{f(p)}Y \longrightarrow \mathbb{R}$



evaluation of the comml nbd of y



$T^* : \mathrm{Smooth} \longrightarrow \mathrm{DLens}(\mathrm{Smooth}) \approx$ reverse derivative

$$\begin{array}{c} X \\ f \downarrow \\ Y \end{array} \longmapsto \begin{tikzcd} T^*X \arrow[d] & X \times_Y T^*Y \arrow[l, "T^*f"'] \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & T^*Y \arrow[d] \\ X \arrow[r, equal] & X \arrow[r, "f"'] & Y \end{tikzcd}$$

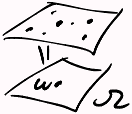
$$T^*f : (x : X) \times (\xi : T_{f(x)}Y) \longrightarrow T_xX$$

(1) What is our feedback like?

~~$R = \mathbb{R}^N$ payoff vector~~ $\qquad R^{\Omega} \cong \Omega \longrightarrow R$

(2) What are our changes? $\qquad P\Omega$

$\omega \in \Omega$

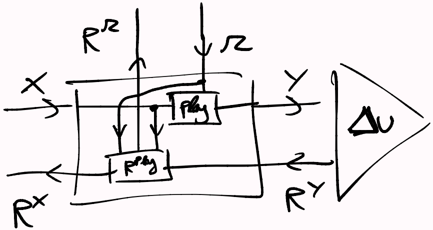


$R^*: \mathbf{Set} \longrightarrow \mathbf{DLens}(\mathbf{Set})$

$$\begin{array}{ccc} X & & X \quad Y \\ f\downarrow & \longmapsto & f\downarrow \quad \uparrow R^f \\ Y & & Y \quad R^Y \end{array} \qquad R^f : X \times R^Y \longrightarrow R^X$$

$$(x, v) \longmapsto f ; v$$

$$X \to Y \to V$$



$$\Delta u : Y \longrightarrow R^Y$$
$$y \longmapsto u$$
$$y \longmapsto \lambda y'.\, u(y') - u(y)$$

Para(R^*)

Para(Set) $\quad (\Omega, f) : X \longrightarrow Y$

$\downarrow \qquad\qquad\qquad \downarrow$

Para(Lens(Set)) $\quad$ Para$(R^*)(\Omega, f)$

lax functor!

$$R^{\Omega \times \Xi} \neq R^\Omega \times R^\Xi$$

$R^*: \mathbf{Set} \longrightarrow \mathbf{DLens}(\mathbf{Set})$ lax monoidal

$$X, Y \longmapsto (X, R^X) \otimes (Y, R^Y) = (X \times Y, R^X \times R^Y)$$

$$\begin{tikzcd} (X \times Y, R^X \times R^Y) \arrow[r, shift left, "1"] & (X \times Y, R^{X \times Y}) \arrow[l, shift left] \end{tikzcd}$$

$$\mu_{X,Y} : X \times Y \times R^{X \times Y} \longrightarrow R^X \times R^Y$$

$$(x, y, v) \longmapsto \langle v(x,-), v(-,y) \rangle$$

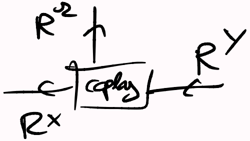
$$(\Omega, f) : X \longrightarrow Y = \Omega \times X \longrightarrow Y$$

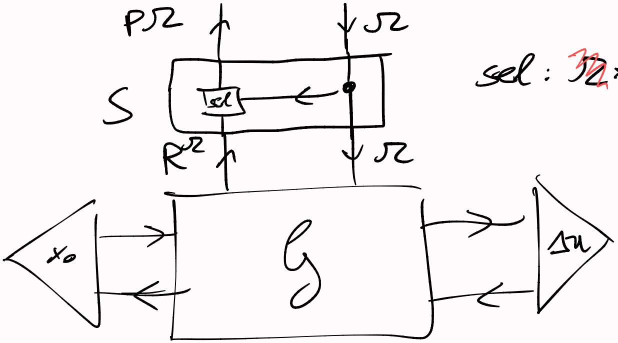
$$\big\downarrow$$

$$R^f : R^Y \longrightarrow R^{\Omega \times X}$$

$$\downarrow$$

$$R^Y \longrightarrow R^\Omega \times R^X$$





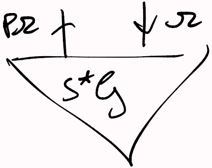
$\mathrm{sel} : \cancel{\Omega} \times R^{\Omega} \longrightarrow P\Omega$

$u \longmapsto$ selection of nice deviations

$u \longmapsto \operatorname{argmax}(u)$

equilibria are fixpoints

$\boxed{\omega \in s^*\mathcal{G}(\omega)}$



$\cong \; \Omega \xrightarrow[s^*\mathcal{G}]{} P\Omega$

Thanks for your attention!

Questions?

References I

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