Polynomial Functors and Shannon Entropy

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TOPOS INSTITUTE

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Outline

Why am I here?

We're here to learn from each other. But what is learning?

- Somehow out of all the information out there, some of it *sticks*.
- We develop frameworks by which to *store* information.
- I'm interested in how intelligence and learning function.
- So I study how knowledge is stored and transferred in databases and...
- ...how dynamical systems interact to adapt and learn (e.g. in DNNs).

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Entropy has been put forward as an approach to intelligence and learning.

- Life can be understood as a *dissipative system*, spraying entropy.
- It does so while packing negentropy—organization—into itself.
- Polani's *empowerment* and Freer's *causal entropic forces*...
- ...are entropy-based approaches to intelligent behavior.

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I only seem to understand things when they're written categorically.

- I've been trying to understand "what entropy really is."
- The Baez-Fritz-Leinster conception of entropy is great,...
- ...but I want to connect it in with dynamical systems or databases.

The overwhelming abundance of Poly

In January 2020 I fell in love with a category called **Poly**.

- Its applications subsume everything I'd done with categ'l databases...
- ...and everything I'd done with interacting dynamical systems.
- It's used in functional programming, type theory, higher cat'y theory.

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- Its applications subsume everything I'd done with categ'l databases...
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- It's used in functional programming, type theory, higher cat'y theory.

But it's not just very applicable, it's also very highly-structured.

- Coproducts and products that agree with usual polynomial arithmetic;
- All limits and colimits;
- At least three orthogonal factorization systems;
- A symmetric monoidal structure $\otimes$ distributing over $+$;
- A cartesian closure q^p and monoidal closure $[p, q]$ for $\otimes$;
- Another nonsymmetric monoidal structure $\triangleleft$ that's duoidal with $\otimes$;
- A left (Meyers?) $\triangleleft$-coclosure $\left[\frac{-}{-}\right]$, meaning $\mathbf{Poly}(p, q \triangleleft r) \cong \mathbf{Poly}(\left[\frac{r}{p}\right], q)$;
- An indexed right $\triangleleft$-coclosure, i.e. $\mathbf{Poly}(p, q \triangleleft r) \cong \sum_{f\colon p(1) \to q(1)} \mathbf{Poly}(p \overset{f}{\frown} q, r)$;
- An indexed right $\otimes$-coclosure (Niu?), i.e. $\mathbf{Poly}(p, q \otimes r) \cong \sum_{f\colon p(1) \to q(1)} \mathbf{Poly}(p \overset{f}{\nearrow} q, r)$;
- At least ten monoidal structures in total;
- $\triangleleft$-monoids generalize Σ-free operads;
- $\triangleleft$-comonoids are exactly categories; bicomodules are data migrations.

See "A reference for categorical structures on **Poly**", arXiv: 2202.00534

Entropy in terms of Poly

Today I'll tell you how entropy looks from the **Poly** point of view.

- I'll show how to think of objects in **Poly** as empirical distributions.
- I'll show that there are distributive monoidal functors

$$\mathbf{Poly}^{\mathbf{Cart}} \xrightarrow{p \mapsto \dot{p}y} \mathbf{Poly} \xrightarrow{p \mapsto (p(1), \Gamma(p))} \mathbf{Set} \times \mathbf{Set}^{\mathrm{op}}$$

  sending $p \in \mathbf{Poly}^{\mathbf{Cart}}$ to an invariant $\hbar(p) := (A, B) \in \mathbf{Set} \times \mathbf{Set}^{\mathrm{op}}$.
- The Shannon entropy can then be extracted: $H(p) = \log(A/\sqrt[A]{B})$.
- Properties of entropy follow from the distributive monoidality of $\hbar$.

Plan

The plan for the rest of the time is as follows:

- Give background on polynomial functors.
- Explain $\hbar\colon \mathbf{Poly}^{\mathbf{Cart}} \to \mathbf{Set} \times \mathbf{Set}^{\mathrm{op}}$ and its relation to entropy.
- Talk about generalizations and future work.
- Conclude.

Outline

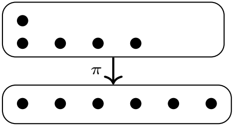
What is a polynomial?

Here's how we'll think about polynomial functors.

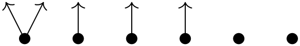
Algebraic

$$y^2 + 3y + 2$$

Bundle



Corolla forest



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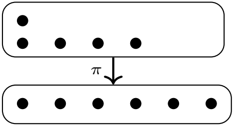

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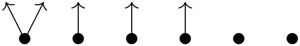


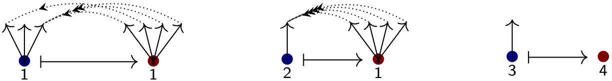
You can think of the bundle as a empirical distribution:

- The first outcome was drawn twice; the next three once; the rest never.
- It corresponds to the distribution $(\frac{2}{5},\ \frac{1}{5},\ \frac{1}{5},\ \frac{1}{5},\ 0,\ 0)$.

What is a morphism of polynomials?

Let $p := y^3 + 2y$ and $q := y^4 + y^3 + y + 1$

A morphism $p \xrightarrow{\varphi} q$ sends p-outcomes to q-outcomes, interpreting draws:



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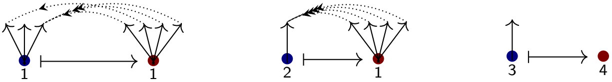


It is *Cartesian* iff each map on draws is a bijection. ($\mathbf{Poly}^{\mathbf{Cart}}(p, q) \cong 6$.)

Fundamental invariants

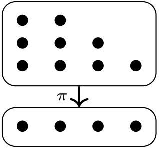
We will be interested in two fundamental invariants of a polynomial.

- From the bundle POV, these would be *base* and *global sections*.
- So if p is represented by $E \to B$, these are B and $\mathbf{Set}_{/B}(B, E)$.
- In terms of polynomials these are

$$p(1) \cong \mathbf{Poly}(y, p) \qquad \text{and} \qquad \Gamma(p) := \mathbf{Poly}(p, y).$$

- E.g. for the following bundle these are $p(1) \cong 4$ and $\Gamma(p) \cong 18$.

$$2y^3 + y^2 + y \quad =$$



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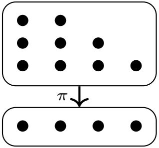

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These are functorial in opposite directions:

$$\mathbf{Poly} \xrightarrow{p \mapsto (p(1), \Gamma(p))} \mathbf{Set} \times \mathbf{Set}^{\mathrm{op}}$$

This functor has a right adjoint, but we won't use that.

The distributive monoidal structure $(\textbf{Poly}, 0, +, y, \otimes)$

The category **Poly** is distributive monoidal.

- The usual sum of two polynomials is their coproduct; 0 is initial.
- There is another operation $\otimes$ called *Dirichlet product*. Formula:

$$p \otimes q := \sum_{(i,j) \in p(1) \times q(1)} y^{p[i] \times q[j]}$$

These are very simple bundle-wise: sum & product of base and total space:

$$\begin{array}{c} E_1 \\ \downarrow \\ B_1 \end{array} + \begin{array}{c} E_2 \\ \downarrow \\ B_2 \end{array} = \begin{array}{c} E_1 + E_2 \\ \downarrow \\ B_1 + B_2 \end{array} \qquad \text{and} \qquad \begin{array}{c} E_1 \\ \downarrow \\ B_1 \end{array} \otimes \begin{array}{c} E_2 \\ \downarrow \\ B_2 \end{array} = \begin{array}{c} E_1 \times E_2 \\ \downarrow \\ B_1 \times B_2 \end{array}$$

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- These clearly distribute: $p \otimes (q_1 + q_2) \cong (p \otimes q_1) + (p \otimes q_2)$.
- Soon we'll see how the fundamental invariants respect these oper'ns.

Derivatives and the bifibration

The derivative of a polynomial functor is another polynomial functor.

- Write $\dot{p}$ for the derivative with respect to y; consider also $\dot{p}y$.

$$\dot{p} = \sum_{i \in p(1)} \sum_{d \in p[i]} y^{p[i] \setminus \{d\}} \qquad \text{and} \qquad \dot{p}y \cong \sum_{i \in p(1)} p[i] y^{p[i]}$$

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Neither of these is functorial in **Poly**, but *they are functorial* in $\textbf{Poly}^{\textbf{Cart}}$.

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The last theory we'll need is the bifibration $\mathbf{Poly} \to \mathbf{Set}$.

- The functor $p \mapsto p(1)$ is a monoidal *-bifibration. All we need:
- For any $f \colon A \to B$, we have $\mathbf{Poly}_A(f^* q, p) \cong \mathbf{Poly}_B(q, f_* p)$.

$$f^* q := \sum_{a \in A} y^{q[fa]} \qquad \text{and} \qquad f_* p := \sum_{b \in B} y^{\sum_{a \mapsto b} p[a]}$$

As we lump outcomes together, we add up the draws.

Outline

$p \mapsto \dot{p}y$ is distributive

We're now ready to get to work on how all this relates to entropy.

- The approach is to extract two invariant sets from any polynomial.
- This process is "good" in that it is a distributive monoidal functor.
- We'll extract the extensive and intensive entropies from these.

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- The derivative is linear, $(p \dot{+} q) = \dot{p} + \dot{q}$, and so is $p \mapsto py$.
- So $p \mapsto \dot{p}y$ preserves coproducts $+$. It also preserves $\otimes$:

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\begin{aligned}
(\dot{p}y) \otimes (\dot{q}y) &\cong \sum_{i \in p(1)} p[i] y^{p[i]} \otimes \sum_{j \in q(1)} q[j] y^{q[j]} \\
&\cong \sum_{(i,j) \in p(1) \times q(1)} p[i] \times q[j] y^{p[i] \times q[j]} \\
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So $(\mathbf{Poly}^{\mathbf{Cart}}, 0, +, y, \otimes) \xrightarrow{p \mapsto \dot{p}y} (\mathbf{Poly}, 0, +, y, \otimes)$ is distributive monoidal.

Fundamental invariants are distributive

The fundamental invariants $p \mapsto (p(1), \Gamma(p))$ are also distributive

$$(\mathbf{Poly}, 0, +, y, \otimes) \xrightarrow{p \mapsto (p(1), \Gamma(p))} (\mathbf{Set} \times \mathbf{Set}^{\mathsf{op}}, (0,1), +, (1,1), \otimes)$$

But what exactly is all this structure on $\mathbf{Set} \times \mathbf{Set}^{\mathsf{op}}$?

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- $\mathbf{Set} \times \mathbf{Set}^{\mathrm{op}}$ has coproducts: $(A_1, B_1) + (A_2, B_2) \cong (A_1 + A_2, B_1 \times B_2)$.
- It has another symmetric monoidal structure with unit $(1,1)$:
$$(A_1, B_1) \otimes (A_2, B_2) := \left(A_1 \times A_2, B_1^{A_2} \times B_2^{A_1}\right)$$
- And these distribute “because” $B^{A_1+A_2}(B_1 B_2)^A \cong (B^{A_1} B_1^A)(B^{A_2} B_2^A)$.

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Why do the fundamental invariants (as a pair) preserve $+$ and $\otimes$?

- We have $(p+q)(1) \cong p(1) + q(1)$ and $\Gamma(p+q) \cong \Gamma(p) \times \Gamma(q)$.
- This says they preserve $+$. One also checks they preserve $\otimes$:

$$(p \otimes q)(1) \cong p(1) \times q(1) \qquad \text{and} \qquad \Gamma(p \otimes q) \cong \Gamma(p)^{q(1)} \times \Gamma(q)^{p(1)}$$

Taking stock

Let's denote the composite of our distributive functors by $\hbar$:

$$\mathbf{Poly}^{\mathbf{Cart}} \xrightarrow{p \mapsto \dot{p}y} \mathbf{Poly} \xrightarrow{p \mapsto (p(1), \Gamma(p))} \mathbf{Set} \times \mathbf{Set}^{\mathrm{op}}$$

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Define a real number $L(A, B) := \frac{\log(\#A^{\#A}) - \log(\#B)}{\#A}$. Then:

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Theorem

Let p be a polynomial, considered as a probability distribution P, and let $H(P)$ be its Shannon entropy. Then we have

$$H(P) = L(\hbar(p))$$

The categorical partition function and entropy

I'm unfamiliar with the thermo picture. Joint with James Dama:

- One should think of Shannon entropy as an *entropy density*.
- The thermo picture defines a *partition function* Ω for distributions.
- For $p \in \mathbf{Poly}$ with $\hbar(p) = (A, B)$, this would be $\Omega_p := \frac{A^A}{B}$.
- Then the extensive entropy of p is given by $\mathrm{E}(p) := \log \Omega(p)$.
- And the Shannon entropy of p is the density $H(p) := \mathrm{E}(p)/A$.

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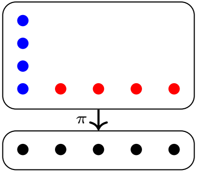
Let's see a worked example. Consider the bundle for $p := y^4 + 4y^1$:



- We find $\dot{p}y = 4y^4 + 4y$, so $\hbar(p) = (4 + 4, 4^4) = (8, 4^4)$.
- So $\Omega_p = \frac{8^8}{4^4}$, Ext've: $\mathrm{E}(p) = \log \Omega_p = 16$, Shannon: $H(p) = 16/8 = 2$.

Consequences of distributivity

The fact that $\hbar$ preserves $\otimes$ and properties of log immediately give us

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The fact that $\hbar$ preserves $+$ is more subtle. We see it better in Ω and E.

- Again joint with James Dama.
- Suppose you write p as a sum, $p := \sum_{a \in A} p_a$.
- This is the same as giving a function $f : p(1) \to A$.
- Recall that from this we get $f_* p \in \mathbf{Poly}_A$, lumping distributions.

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It follows from the fact that $\hbar$ preserves sums that:

$$\Omega_p = \Omega_{f_* p} \times \prod_{a \in A} \Omega_{p_a} \qquad \text{and} \qquad \mathrm{E}(p) = \mathrm{E}(f_* p) + \sum_{a \in A} \mathrm{E}(p_a)$$

The usual "chain rule" for Shannon entropy follows directly from this.

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- **Poly** is the most highly-structured category I've ever seen.
- Both have applications to interacting dynamical systems.

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Entropy still feels somehow foreign to me.

- Hopefully, having different categorifications will help clarify it.
- I still have hope that it will someday bond with the dynamics of **Poly**.

Thanks! Comments and questions welcome...