

Cornering Optics

Chad Nester

Tallinn University of Technology

(with Guillaume Boisseau and Mario Román)

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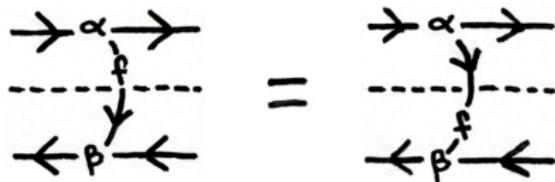
Optics $\langle \alpha \mid \beta \rangle_M : (A, B) \rightarrow (C, D)$ in a monoidal category \mathbb{A} consist of arrows $\alpha : A \rightarrow M \otimes C$ and $\beta : M \otimes D \rightarrow B$ in \mathbb{A} .



For example, *lenses* are optics in a cartesian monoidal category.

Optics are subject to “sliding equations” of the following form:

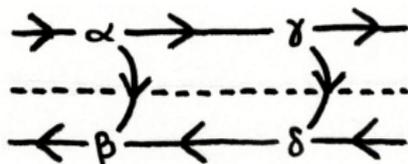
$$\langle \alpha(f \otimes 1_C) \mid \beta \rangle_N = \langle \alpha \mid (f \otimes 1_C)\beta \rangle_M$$



where $f : M \rightarrow N$, $\alpha : A \rightarrow M \otimes C$, $\beta : N \otimes D \rightarrow B$ in \mathbb{A} .

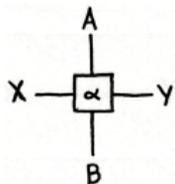
Optics in \mathbb{A} form a category $\text{Optic}_{\mathbb{A}}$. Composition is given by:

$$\langle \alpha \mid \beta \rangle_M \langle \gamma \mid \delta \rangle_N = \langle \alpha(1_M \otimes \gamma) \mid (1_M \otimes \delta)\beta \rangle_{M \otimes N}$$

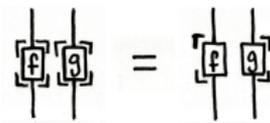
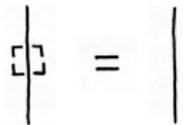
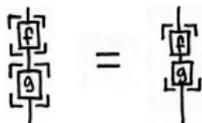
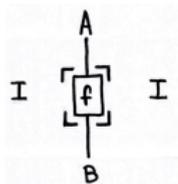


and identities are given by $\langle 1_A \mid 1_A \rangle_I : (A, A) \rightarrow (A, A)$.

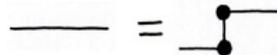
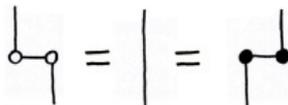
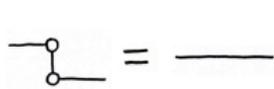
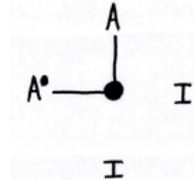
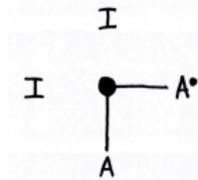
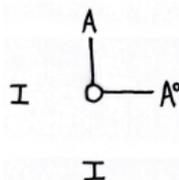
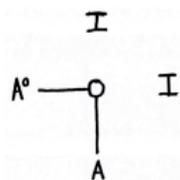
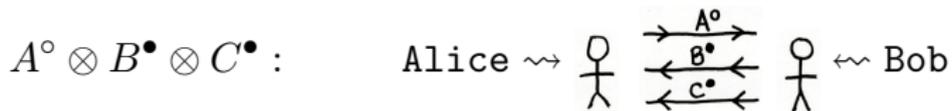
We construct a single object double category \mathbb{A} .



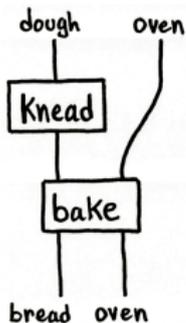
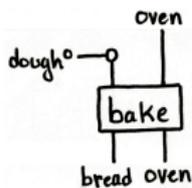
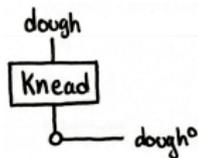
The horizontal edge monoid of \mathbb{A} is $(\mathbb{A}_0, \otimes, I)$.



The vertical edge monoid of $[\mathbb{A}]$ is $(\mathbb{A}_0 \times \{\circ, \bullet\})^*$.

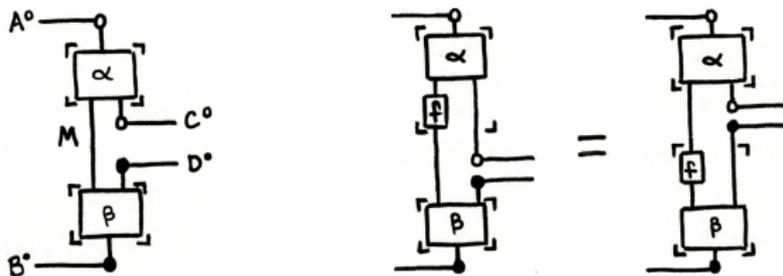
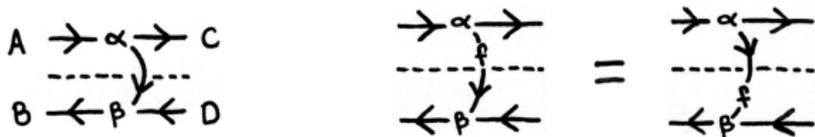


Cells of $[\mathbb{A}]$ are like arrows of \mathbb{A} that interact with the environment.



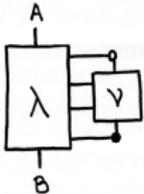
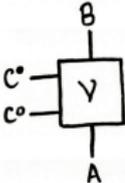
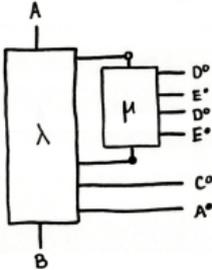
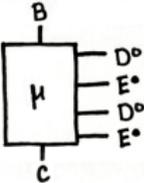
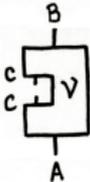
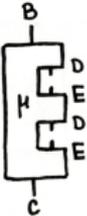
When composed horizontally, they interact with each other.

$\text{Optic}_{\mathbb{A}}$ embeds into $\mathbf{H}[\mathbb{A}]$:



In fact, $\text{Optic}_{\mathbb{A}}$ is the full subcategory of $\mathbf{H}[\mathbb{A}]$ on objects $A^\circ \otimes B^\circ$.

Exotic operations on comb diagrams:



References:

Cornering Optics

Guillaume Boisseau, Chad Nester, and Mario Román
ACT 2022

The Structure of Concurrent Process Histories

Chad Nester
COORDINATION 2021

Situated Transition Systems

Chad Nester
ACT 2021

Open Diagrams via Coend Calculus

Mario Román
ACT 2020