Characterization of Contextuality with Semi-Module Čech Cohomology
(arXiv:2104.11411)

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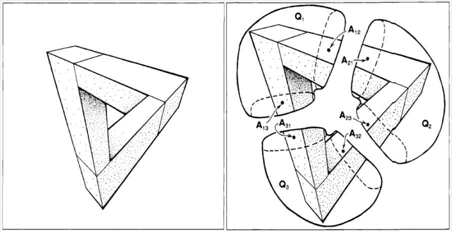

Sheaves and contextuality[1]

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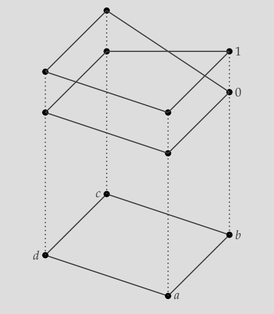

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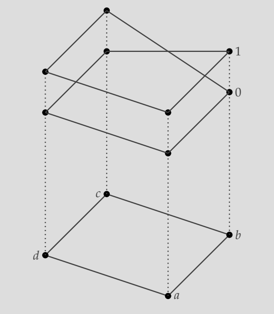

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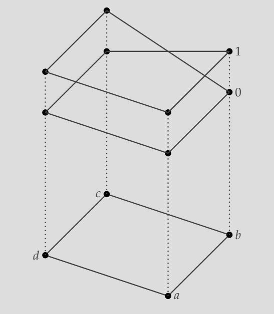

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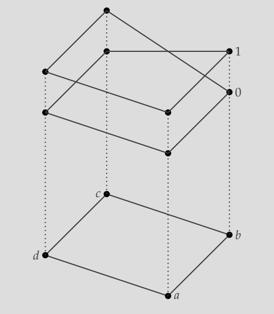

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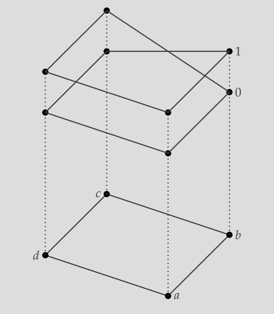

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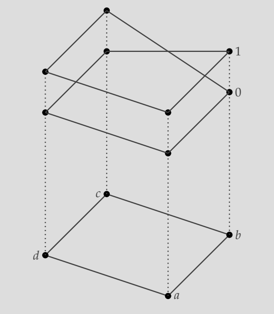

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An empirical model is non-contextual if

$$\mu_R^{O^U}(A) = \sum_{\Lambda} p(\lambda)\,\xi(A|\lambda) = \sum_{\Lambda} p(\lambda) \prod_{x \in U} \mu_R^{O^x}(\rho'(U,x)(A), \lambda).$$

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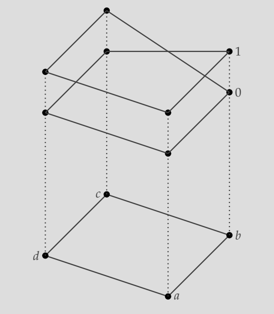

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Theorem (Fine-Abramsky-Brandenburger)

If ξ is deterministic (outcome-determinism), then the hidden variables λ can be seen as the global sections of $\mathcal{E}$.

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- Complex of extended cochains

$$\mathbf{0} \longrightarrow C^0(\mathcal{C}, \mathcal{F}) \xrightarrow{d^0} C^1(\mathcal{C}, \mathcal{F}) \xrightarrow{d^1} C^2(\mathcal{C}, \mathcal{F}) \xrightarrow{d^2} \dots \tag{1}$$

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- $\check{H}^q(\mathcal{C}, \mathcal{F}) = Z^q(\mathcal{C}, \mathcal{F}) / B^q(\mathcal{C}, \mathcal{F}) = \ker(d^q) / \mathrm{Im}(d^{q-1})$

Cohomological contextuality[2]

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[2] Samson Abramsky, Shane Mansfield Rui Soares Barbosa (2012): *The Cohomology of Non-Locality and Contextuality*

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Let $\mathcal{C}$ be connected, $C_{j_0} \in \mathcal{C}$, and $r_{j_0} \in \mathcal{F}(C_{j_0})$. Thus $\gamma(r_{j_0}) = 0$ iff there is a compatible family $\{s_{j_k} \in \mathcal{F}(C_{j_k})\}_{C_{j_k} \in \mathcal{C}}$ such that $r_{j_0} = s_{j_0}$.

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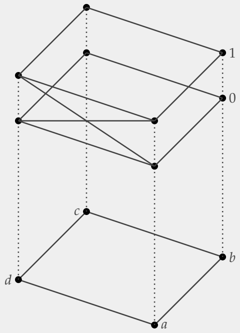

	00	01	10	11
ab	$1_{\mathbb{B}}$	0	0	$1_{\mathbb{B}}$
bc	$1_{\mathbb{B}}$	0	0	$1_{\mathbb{B}}$
bc	$1_{\mathbb{B}}$	0	0	$1_{\mathbb{B}}$
da	$1_{\mathbb{B}}$	$1_{\mathbb{B}}$	$1_{\mathbb{B}}$	$1_{\mathbb{B}}$

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Cohomology with semi-modules[3]

Semi-modules C^q and d_+^q, d_-^q satisfying $d_+^{q+1} \circ d_+^q + d_-^{q+1} \circ d_-^q = d_-^{q+1} \circ d_+^q + d_+^{q+1} \circ d_-^q$

$$\cdots \underset{d_-^{q-2}}{\overset{d_+^{q-2}}{\rightrightarrows}} C^{q-2} \underset{d_-^{q-1}}{\overset{d_+^{q-1}}{\rightrightarrows}} C^q \underset{d_-^{q}}{\overset{d_+^{q}}{\rightrightarrows}} C^{q+1} \underset{d_-^{q+1}}{\overset{d_+^{q+1}}{\rightrightarrows}} \cdots \tag{2}$$

$Z^q = \left\{c \in C^q | d_+^q(c) = d_-^q(c)\right\}$ e $H^q(\mathcal{C}) = Z^q(\mathcal{C}) / \sim^q$.

[3] Alex Patchkoria (2006): *On exactness of long sequences of homology semimodules*

[4] Jaiung Jun (2017): *Čech cohomology of semiring schemes*

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Čech cohomology with semi-modules[4]

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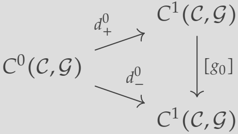
- $g[\sigma]d^n_+(c)(\sigma) = d^n_-(c)(\sigma)$;



Obstruction

- $g[\sigma]d_+^n(c)(\sigma) = d_-^n(c)(\sigma)$;
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$$\begin{array}{ccc} & & C^1(\mathcal{C},\mathcal{G}) \\ & \overset{d_+^0}{\nearrow} & \\ C^0(\mathcal{C},\mathcal{G}) & & \downarrow [g_0] \\ & \underset{d_-^0}{\searrow} & \\ & & C^1(\mathcal{C},\mathcal{G}) \end{array}$$

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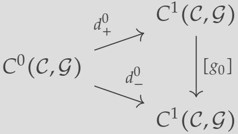

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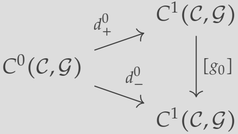

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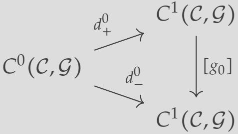

Relative cohomology

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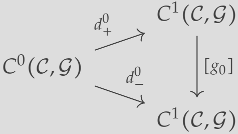

Relative cohomology

- $0 \longrightarrow \mathcal{G}_{\bar{C}} \longrightarrow \mathcal{G} \xrightarrow{p} \mathcal{G}|_C$;

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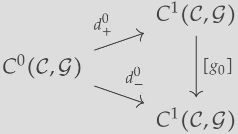

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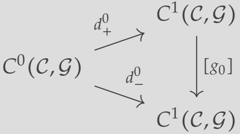

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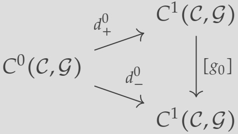

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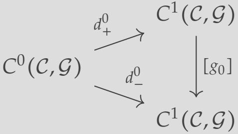

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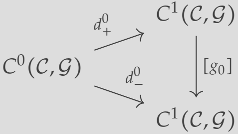

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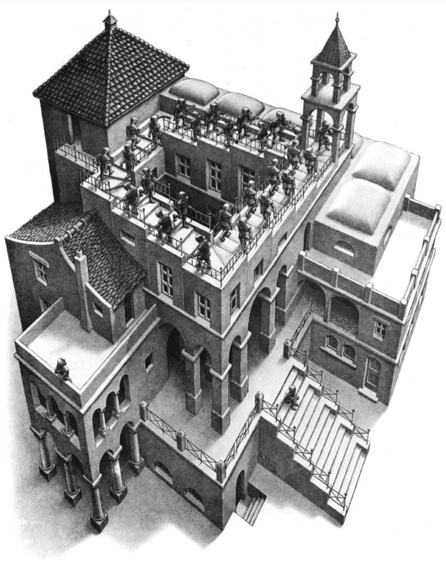
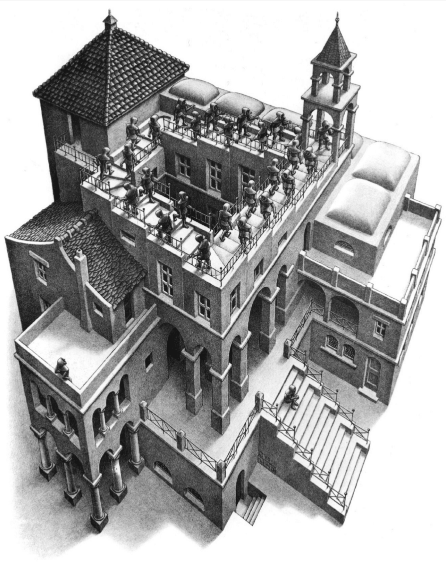
Corollary (Characterization)

An empirical model is contextual iff there is a section of $\mathcal{G}$ with non-trivial obstruction.





Thank you