String Diagrams for Layered Explanations

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Explanations and levels of abstraction

Motivation

Ability to provide an explanation is increasingly important in various areas of (computer) science:

[1] Krivine, 2017
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1. Functional vs. mechanistic level
2. What level of detail is wanted, who the explanation is given to etc.

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Layers of abstraction

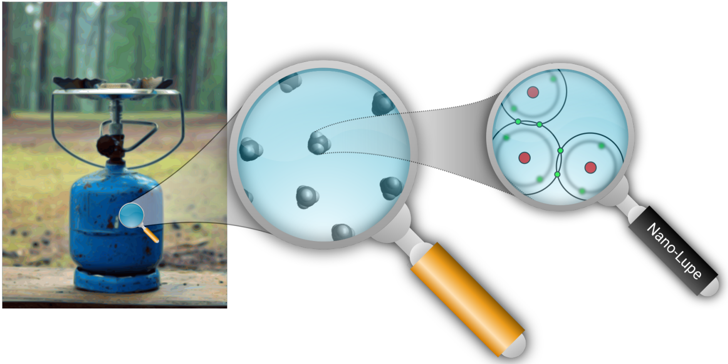

Image source: Openclipart

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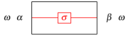
- What is being explained is seen as a *process*
- Processes and their explanations should *compose*
- Modularity: an explanation of a sub-process should be seen as a partial explanation of the larger process

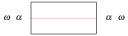
Layered props

$$\frac{\sigma \in \Sigma \setminus \Sigma^i}{\sigma : (\mathtt{ar}(\sigma) \mid \mathtt{coar}(\sigma))}$$

$$\sigma \in \Sigma^i \quad \mathtt{ar}(\sigma) = \omega, \alpha \quad \mathtt{coar}(\sigma) = \omega, \beta$$



$: (\omega, \alpha \mid \omega, \beta)$



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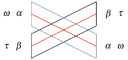
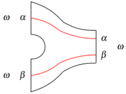


$: (\omega, \alpha; \tau, \beta \mid \tau, \beta; \omega, \alpha)$

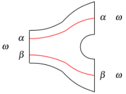
$: (\varepsilon \mid \varepsilon)$

$: (\varepsilon \mid \omega, \varepsilon)$

$: (\omega, \varepsilon \mid \varepsilon)$

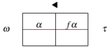


$: (\omega, \alpha; \omega, \beta \mid \omega, \alpha\beta)$



$: (\omega, \alpha\beta \mid \omega, \alpha; \omega, \beta)$

$$f : \omega \to \tau$$



$: (\omega, \alpha \mid \tau, f\alpha)$

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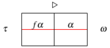

$: (\tau, f\alpha \mid \omega, \alpha)$

$$\frac{x : (t \mid s) \quad y : (s \mid u)}{x ; y : (t, u)} \qquad \frac{x : (\omega, \alpha \mid \omega, \gamma) \quad y : (\omega, \beta \mid \omega, \delta)}{x \otimes_\omega y : (\omega, \alpha\beta \mid \omega, \gamma\delta)} \qquad \frac{x : (t \mid s) \quad y : (u \mid w)}{x \otimes y : (t ; u \mid s ; w)}$$

Example: glucose phosphorylation

after Jean Krivine

We wish to explain the chemical reaction rule:

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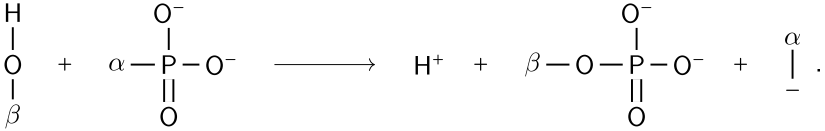

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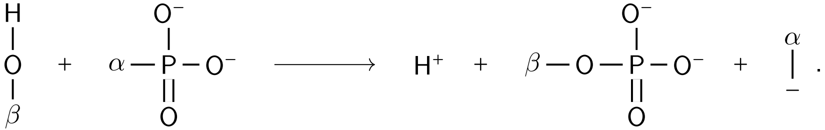

We draw this as an internal box:

in the layered prop.

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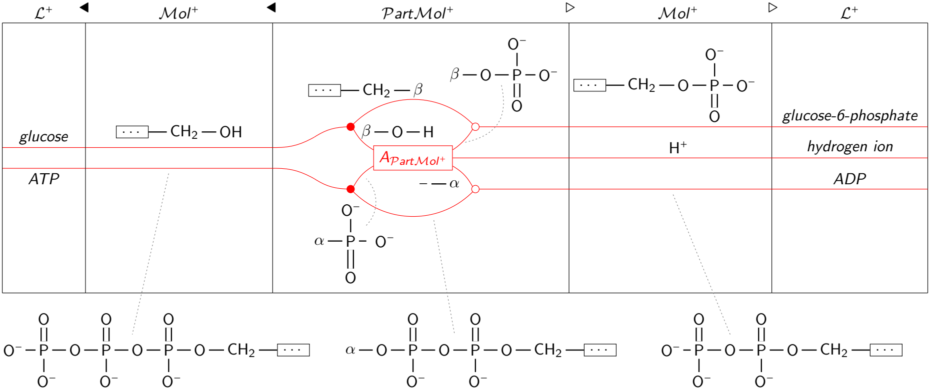
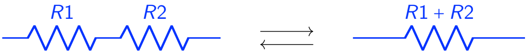


Example: electrical circuits

after Boisseau and Sobociński

We wish to explain the rule for the sequential composition of resistors:



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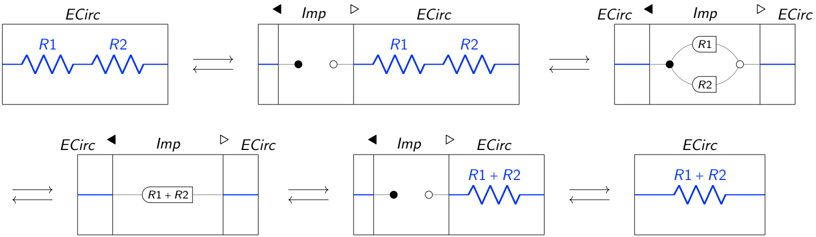

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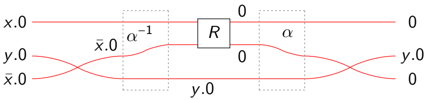

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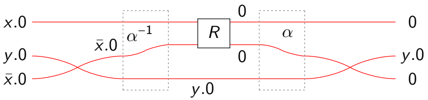
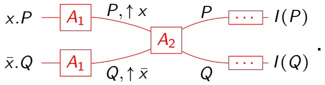


which can be turned into an explanation using the translation:



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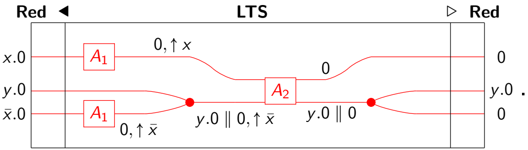
There is also a counterfactual explanation:



Definition (Explanation of a 1-cell)

Let $\mathfrak{e}$ and σ be parallel 1-cells in a layered prop. We say that $\mathfrak{e}$ is an *explanation* of σ if

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3. there is either a 2-cell $\mathfrak{e} \to \sigma$ or a 2-cell $\sigma \to \mathfrak{e}$.

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Formal definitions

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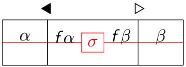
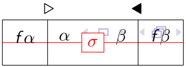
Definition (Window, cowindow)

A *window* is a morphism in a layered prop of the form on the left below. Dually, a *cowindow* is a morphism in a layered prop of the form on the right below.





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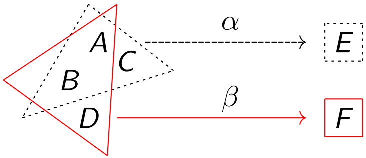

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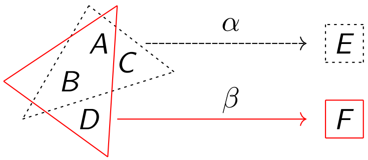

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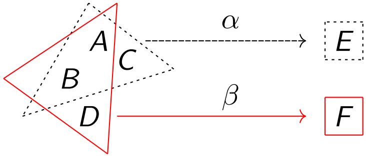

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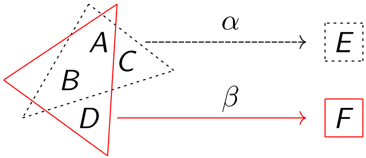

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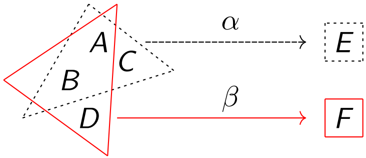

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- Other ideas welcome!

References

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- Jean Krivine. *Physical systems, composite explanations and diagrams*. SYCO 5, 2019.
- Samer B. Nashed, Saaduddin Mahmud, Claudia V. Goldman, Shlomo Zilberstein. *A Unifying Framework for Causal Explanation of Sequential Decision Making*. arXiv:2205.15462v1 2022.
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Thank you for your attention!