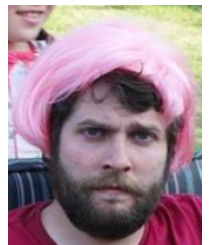
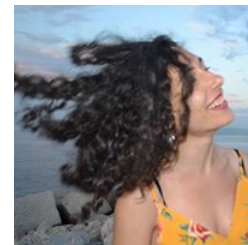
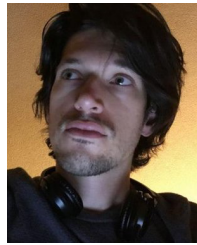
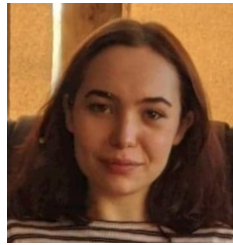
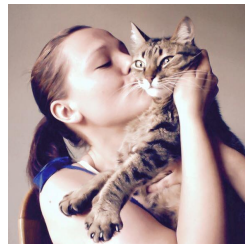
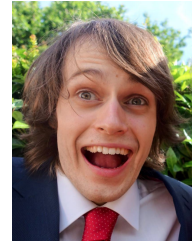
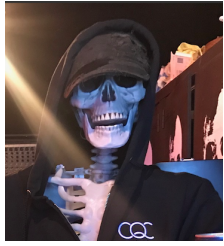


Bob Coecke

## Quantinuum - Compositional intelligence

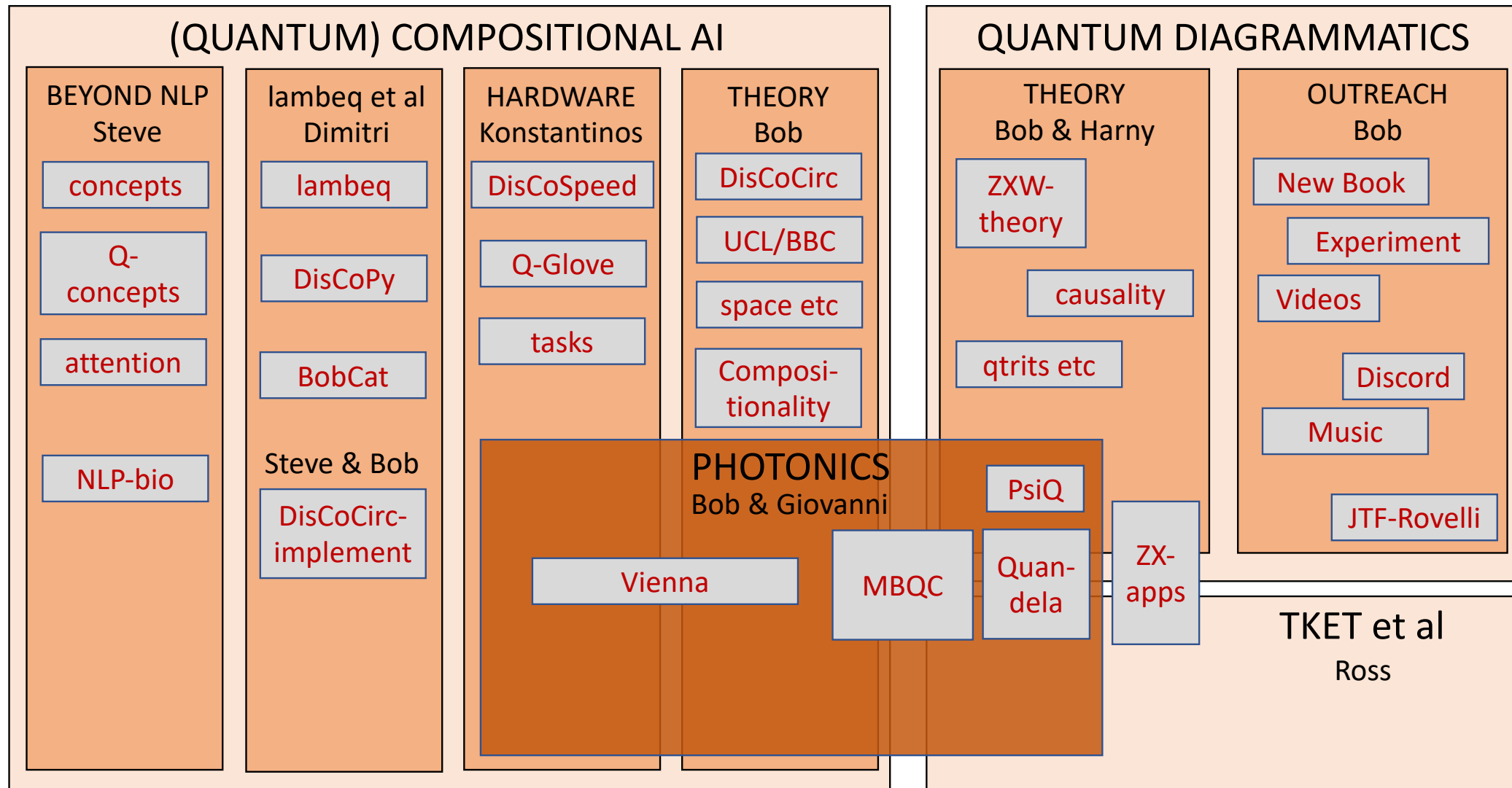


# Oxford team





# (QUANTUM) COMPOSITIONAL INTELLIGENCE



## Questions:

- What is compositionality?



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## Questions:

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  - Practical compositional AI



# COMPOSITIONALITY

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most notably of a category-theoretic origin,  
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what is compositionality?

there are different uses of the term!



arXiv.org > math > arXiv:2110.05327

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## Mathematics > Category Theory

[Submitted on 11 Oct 2021]

# Compositionality as we see it, everywhere around us

Bob Coecke

There are different meanings of the term "compositionality" within science: what one researcher would call compositional, is not at all compositional for another researcher. The most established conception is usually attributed to Frege, and is characterised by a bottom-up flow of meanings: the meaning of the whole can be derived from the meanings of the parts, and how these parts are structured together.

Inspired by work on compositionality in quantum theory, and categorical quantum mechanics in particular, we propose the notions of Schrodinger, Whitehead, and complete compositionality. Accounting for recent important developments in quantum technology and artificial intelligence, these do not have the bottom-up meaning flow as part of their definitions.

Schrodinger compositionality accommodates quantum theory, and also meaning-as-context. Complete compositionality further strengthens Schrodinger compositionality in order to single out theories like ZX-calculus, that are complete with regard to the intended model. All together, our new notions aim to capture the fact that compositionality is at its best when it is `real', `non-trivial', and even more when it also is `complete'.

At this point we only put forward the intuitive and/or restricted formal definitions, and leave a fully comprehensive definition to future collaborative work.

**Frege compositionality** in formal linguistics:

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Meaning of a whole (cf. sentence) should only depend on meanings of its parts (cf. words) and how they are fitted together (cf. grammar).

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Never ask for word meaning in isolation, but only in the context of a sentence.

## **Frege compositionality** in formal linguistics:

Meaning of a whole (cf. sentence) should only depend on meanings of its parts (cf. words) and how they are fitted together (cf. grammar).

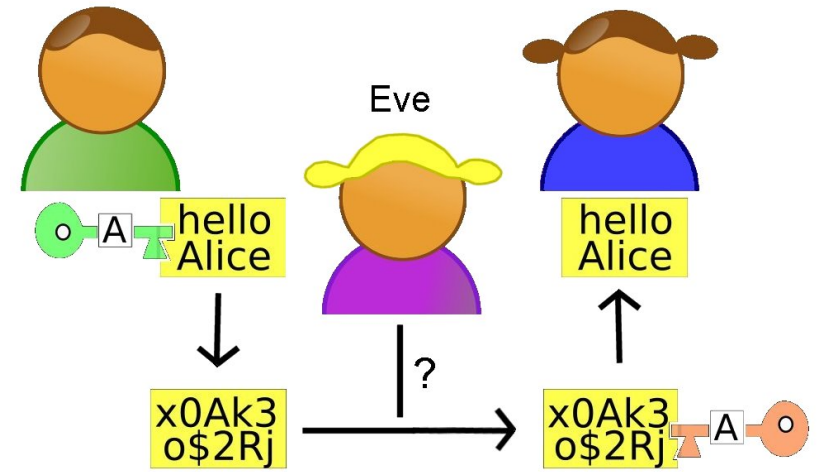
⇒ bottom-up meaning flow

But there is also **Frege's context principle**:

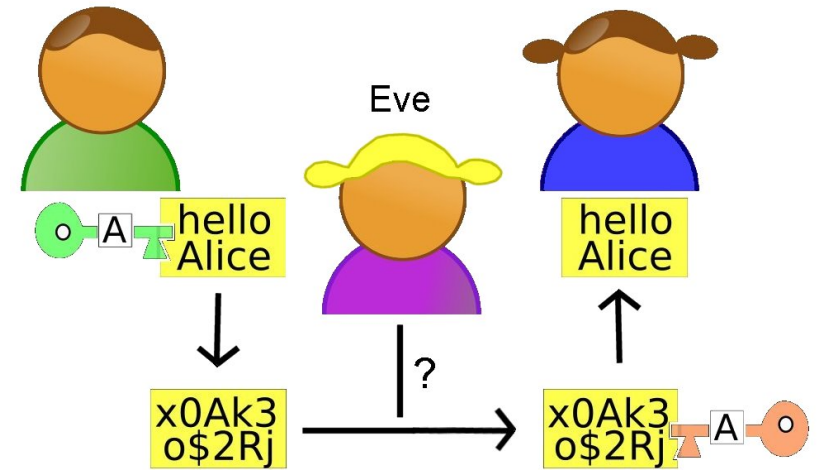
Never ask for word meaning in isolation, but only in the context of a sentence.

⇒ top-down meaning flow

These Alice's get easily disambiguated by context:



These Alice's get easily disambiguated by context:



These Alice's get less easy disambiguated by context:

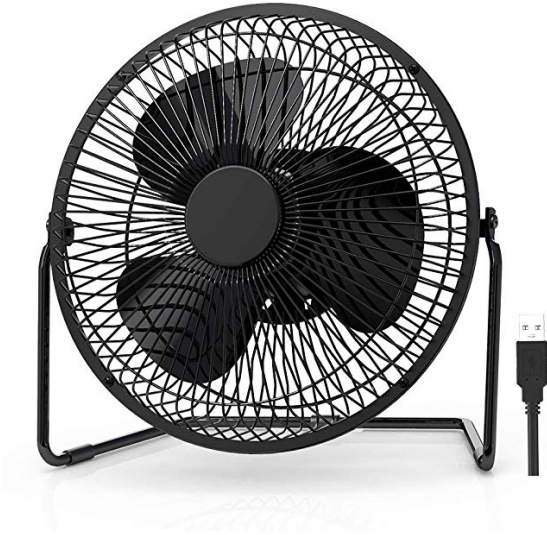


The ambiguity can also intertwine grammar and meaning:

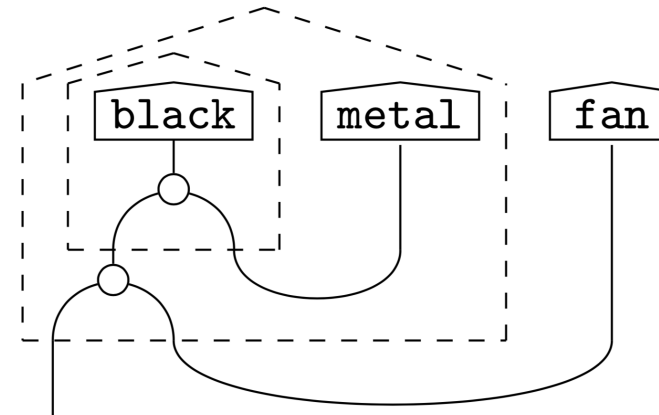
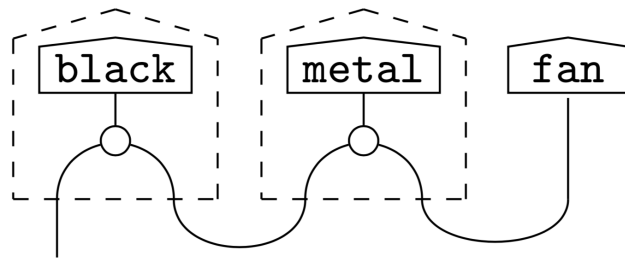




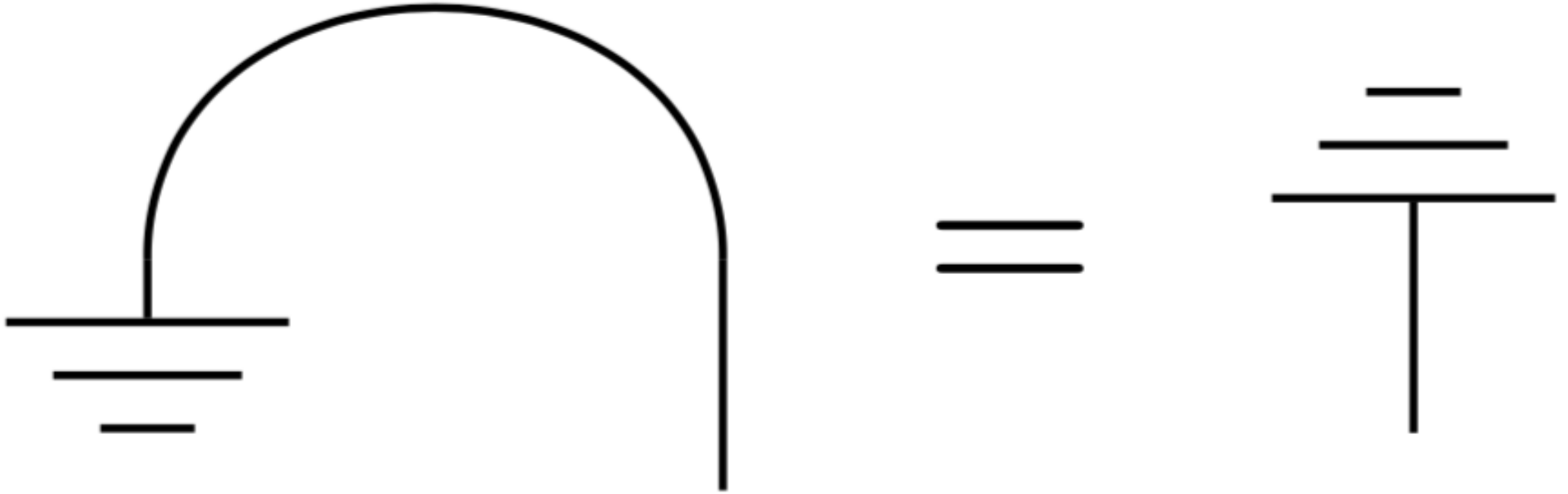
The ambiguity can also intertwine grammar and meaning:



Respectively:



In quantum the situation is even worse e.g. Bell-state:



# quantum industry use of quantum pictorialism

circuit optim



optical MBQC



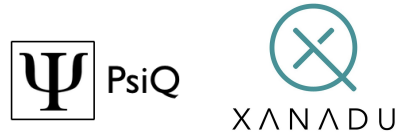
error-corr/surg



q-crypto



other



pro education



mass education

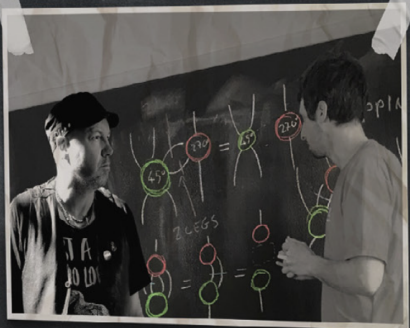


quantum AI



chem/bio





Bit quodit, inverte exeri opta sum  
sapiduiatios etures eaque autem  
dorit slonos et inctiur?

Imus, earum con reptasi niscianda quiatatat.  
Tibusa commo offic tentia conserum dolut lam  
repellab in et fugitat iaercit, omnis dis eat volum  
sam vendi deseque rae modia dellend iorep.



QUANTUM IN PICTURES

QUANTUM IN PICTURES



# QUANTUM IN PICTURES



BOB  
COECKE & STEFANO  
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"Picturing Quantum Processes is a lively and refreshing range through the author's diagrammatic and categorical approach to quantum processes. I recommend this book with no lower age limit required!"  
—Isak Kaufman, University of Illinois

"This book develops from scratch the category-theoretic and diagrammatic language for quantum theory, especially quantum processes. It is a remarkable achievement, vigorous, crystal-clear, complete, and a delight to read."  
—Jeremy Butterfield, University of Cambridge

The unique features of the quantum world are explained in this book through the language of diagrams, setting out an innovative visual method for presenting complex theories. Requiring only basic mathematical literacy, this book employs a unique formalism that builds an intuitive understanding of quantum systems while eliminating the need for complex calculations. This entirely diagrammatic presentation of quantum theory represents the culmination of 10 years of research, uniting classical techniques in linear algebra and Hilbert spaces with cutting-edge developments in quantum computation and foundations.

Written in an entertaining and user-friendly style and including more than 100 exercises, this book is an ideal first course in quantum theory, foundations, and computation for students from undergraduate to PhD level, as well as an opportunity for researchers from a broad range of fields, from physics to biology, linguistics, and cognitive science, to discover a new set of tools for studying processes and interaction.

Bob Coecke is Professor of Quantum Foundations, Logic and Structures at Oxford University, where he also heads the multi-disciplinary Quantum Group. His pioneering research stretches from categorical quantum mechanics to the computational structure of natural language meaning, and recent interests include causality and cognitive architecture.

Aleks Kissinger is an Assistant Professor of Quantum Structures and Logic at Radboud University. His research focuses on diagrammatic language, rewrite theory, category theory, and applications to quantum computation and the foundations of physics.

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# PICTURING QUANTUM PROCESSES

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A First Course in  
Diagrammatic Reasoning

BOB COECKE AND  
ALEKS KISSINGER



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# PICTURING QUANTUM PROCESSES

A First Course in Quantum Theory and  
Diagrammatic Reasoning

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– *process theory (from dodo-book)* –

A *process theory* consists of:

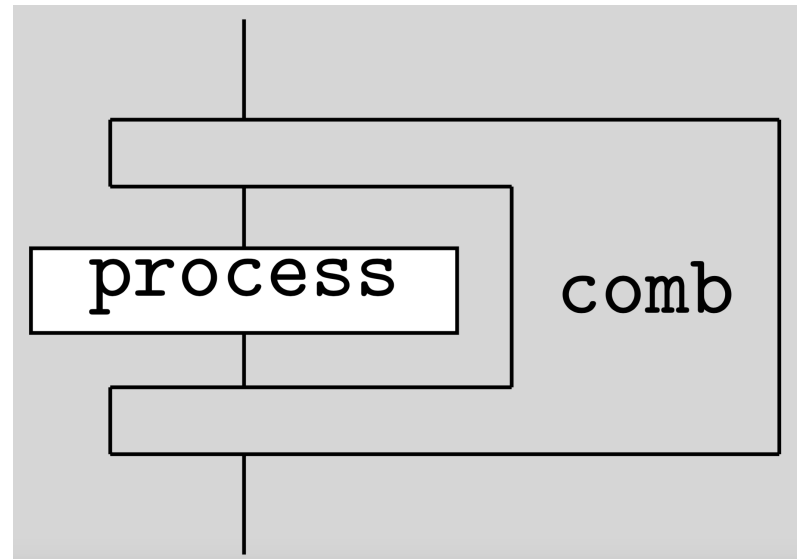
- systems  $S$  represented by wires,
- processes  $P$  represented by boxes, with systems in  $S$  as inputs/outputs,
- composition of processes, represented by wirings, resulting in a process  $D$ .

– *process theory (from dodo-book)* –

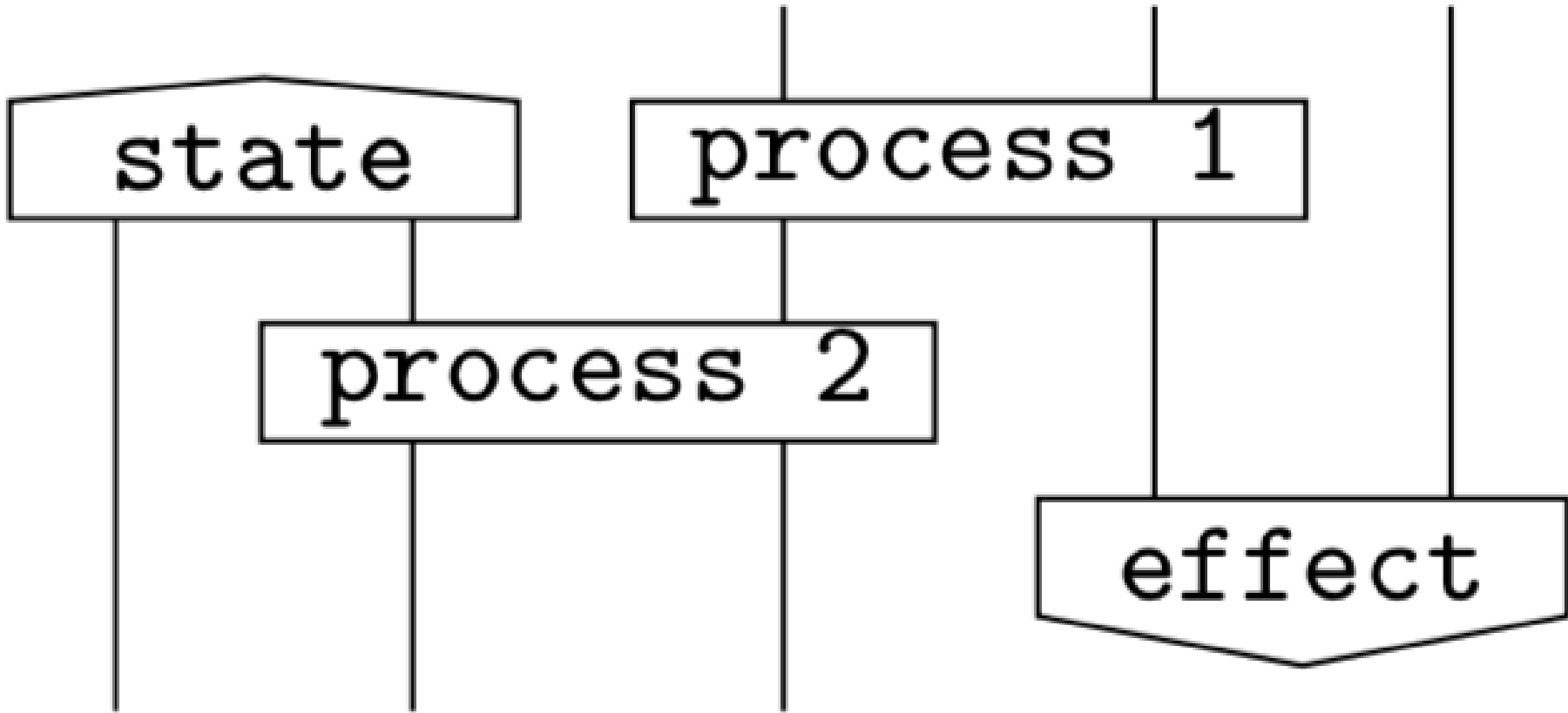
A *process theory* consists of:

- systems  $S$  represented by wires,
- processes  $P$  represented by boxes, with systems in  $S$  as inputs/outputs,
- composition of processes, represented by wirings, resulting in a process  $D$ .

Could be generalised further e.g.:

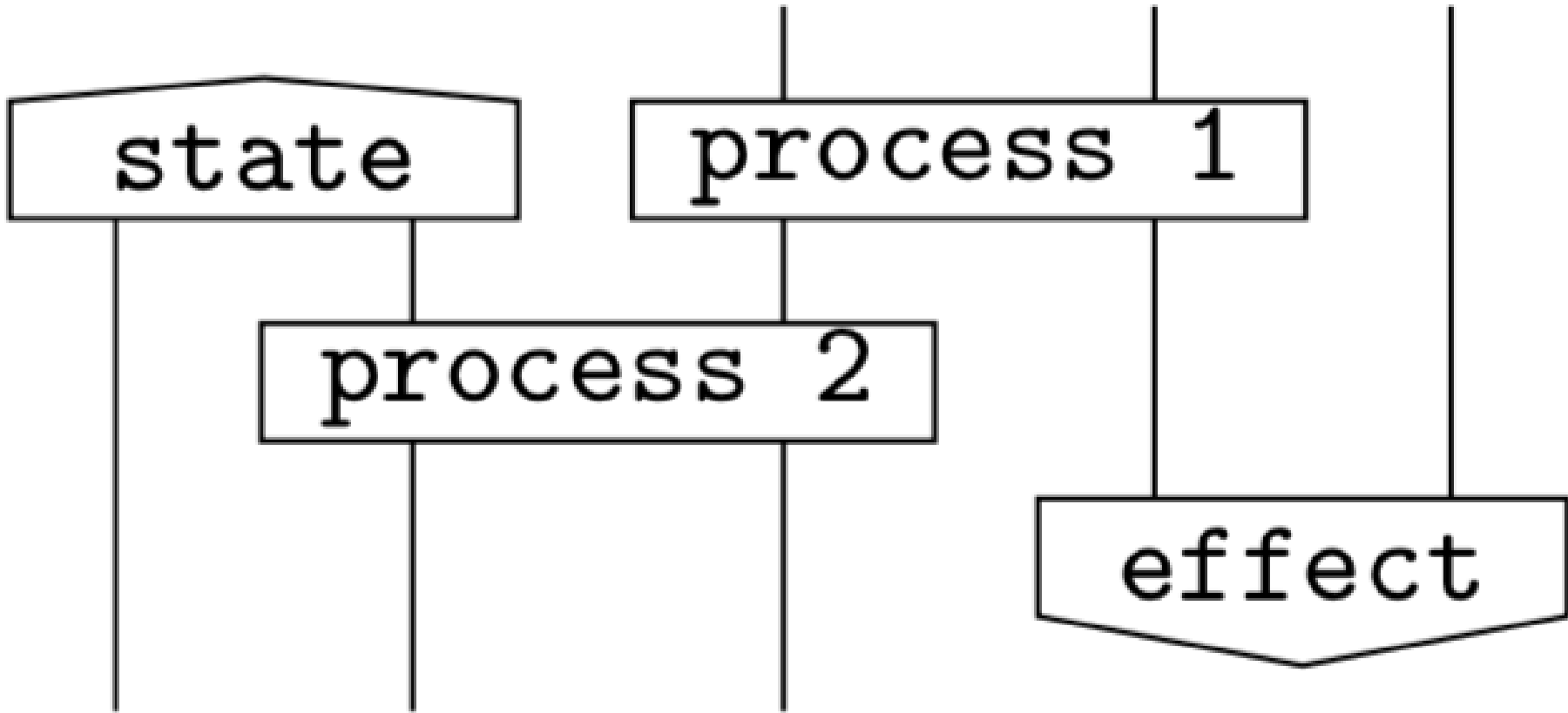


A **Schrödinger compositional theory** is a process theory with diagrams:



such that:

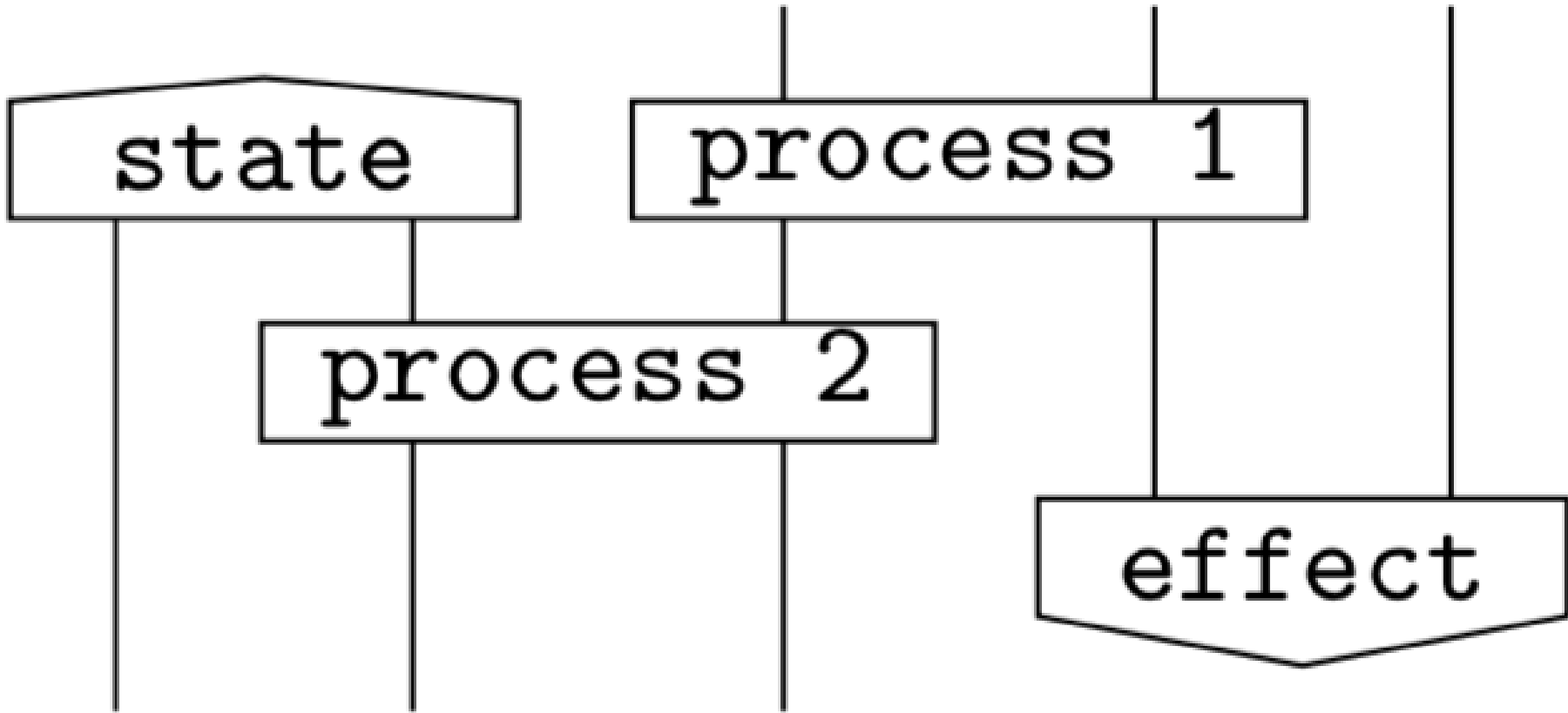
A **Schrödinger compositional theory** is a process theory with diagrams:



such that:

- Composition is non-trivial, i.e. a whole cannot be decomposed meaningfully.

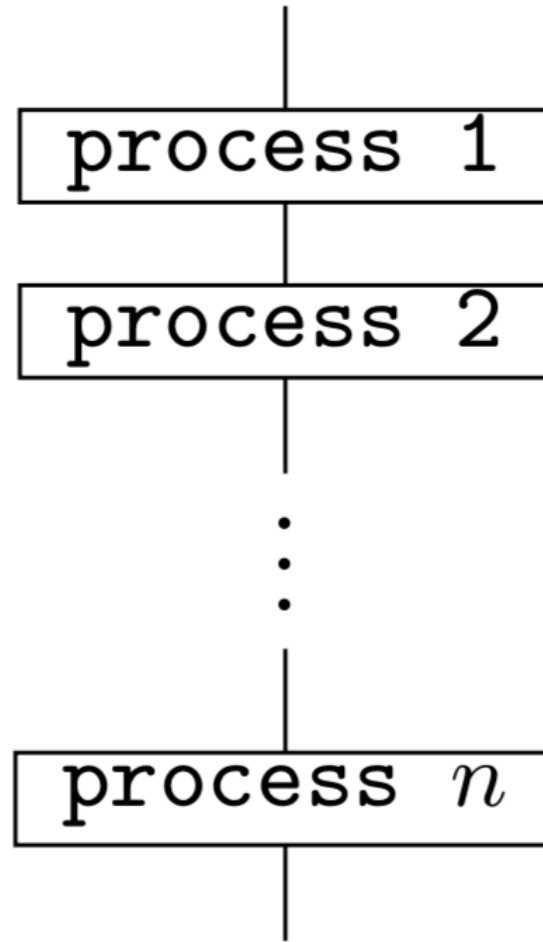
A **Schrödinger compositional theory** is a process theory with diagrams:



such that:

- Composition is non-trivial, i.e. a whole cannot be decomposed meaningfully.
- All ingredients have clear meaningful ontological counterparts in reality.

**Whitehead-compositional theory** is a process theory with diagrams:

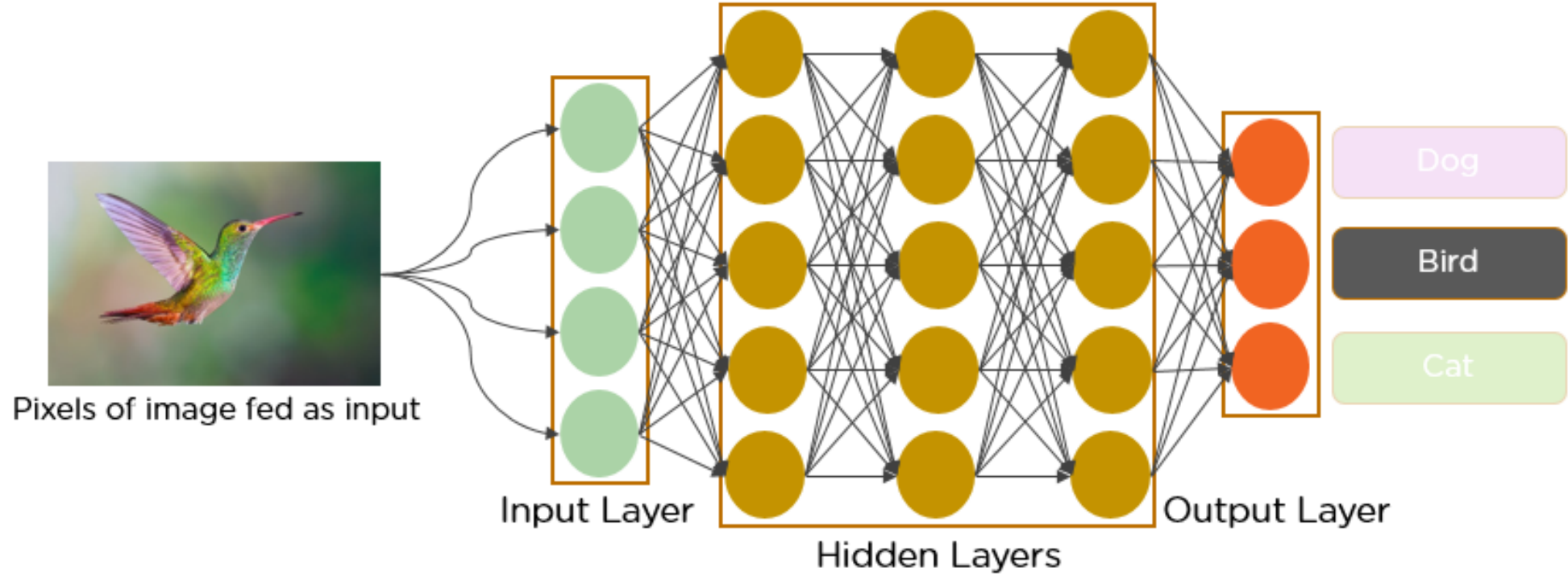


such that:

- All ingredients have clear meaningful ontological counterparts in reality.



non-example:



# ZX-calculus, 2007



## A graphical calculus for quantum observables

Bob Coecke and Ross Duncan  
*Oxford University Computing Laboratory*

We present novel laws describing the interaction of a pair of mutually unbiased observables. These laws yield a diagrammatic calculus which enables matrix-free reasoning about quantum systems. To illustrate the elegance of this approach we establish some properties of standard quantum logic gates, compute the quantum Fourier transform and demonstrate equivalence between certain cluster state and quantum circuit computations.

$$\begin{array}{c} \cdots \\ \vdots \\ \alpha \end{array} \begin{array}{c} \cdots \\ \vdots \\ \beta \end{array} = \begin{array}{c} \cdots \\ \vdots \\ \alpha + \beta \end{array} \quad (\text{S1})$$

$$\begin{array}{c} \bullet \\ | \end{array} = \begin{array}{c} \bullet \\ | \end{array} = \begin{array}{c} | \end{array} \quad (\text{S2})$$

$$\begin{array}{c} \bullet \\ | \\ \bullet \\ \vdots \end{array} = \begin{array}{c} \bullet \\ | \end{array} \begin{array}{c} \bullet \\ | \end{array} \quad (\text{B1})$$

$$\begin{array}{cc} \bullet & \bullet \\ | & | \\ \bullet & \bullet \end{array} = \begin{array}{c} \bullet \\ | \\ \bullet \end{array} \quad (\text{B2})$$

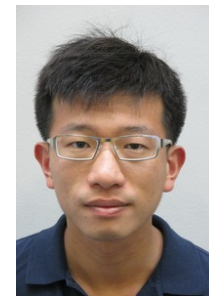
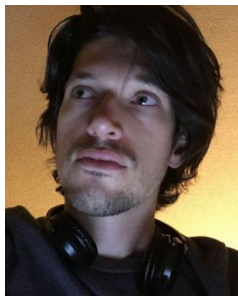
$$\begin{array}{c} H \\ | \end{array} = \begin{array}{c} \bullet \\ | \\ \bullet \\ | \\ \bullet \end{array} \quad (\text{H1})$$

$$\begin{array}{c} \cdots \\ \vdots \\ \bullet \end{array} = \begin{array}{c} H \cdots H \\ \vdots \\ \bullet \\ \vdots \\ H \cdots H \end{array} \quad (\text{H2})$$

$$\begin{array}{c} \bullet \\ | \\ \bullet \end{array} = \begin{array}{c} \bullet \\ | \\ \bullet \end{array} \quad (\text{N})$$

$$\begin{array}{c} \alpha_1 \\ \beta_1 \\ \gamma_1 \end{array} \stackrel{(*)}{=} \begin{array}{c} \alpha_2 \\ \beta_2 \\ \gamma_2 \end{array} \quad (\text{P})$$

Everything!



# Two complete axiomatisations of pure-state qubit quantum computing

**Authors:**  [Amar Hadzihasanovic](#),  [Kang Feng Ng](#),  [Quanlong Wang](#) [Authors Info & Claims](#)

LICS '18: Proceedings of the 33rd Annual ACM/IEEE Symposium on Logic in Computer Science • July 2018 • Pages 502–511 • <https://doi.org/10.1145/3209108.3209128>

**Online:** 09 July 2018 [Publication History](#)

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# Interacting Bialgebras Are Frobenius

Filippo Bonchi<sup>1</sup>, Paweł Sobociński<sup>2</sup>, and Fabio Zanasi<sup>1</sup>

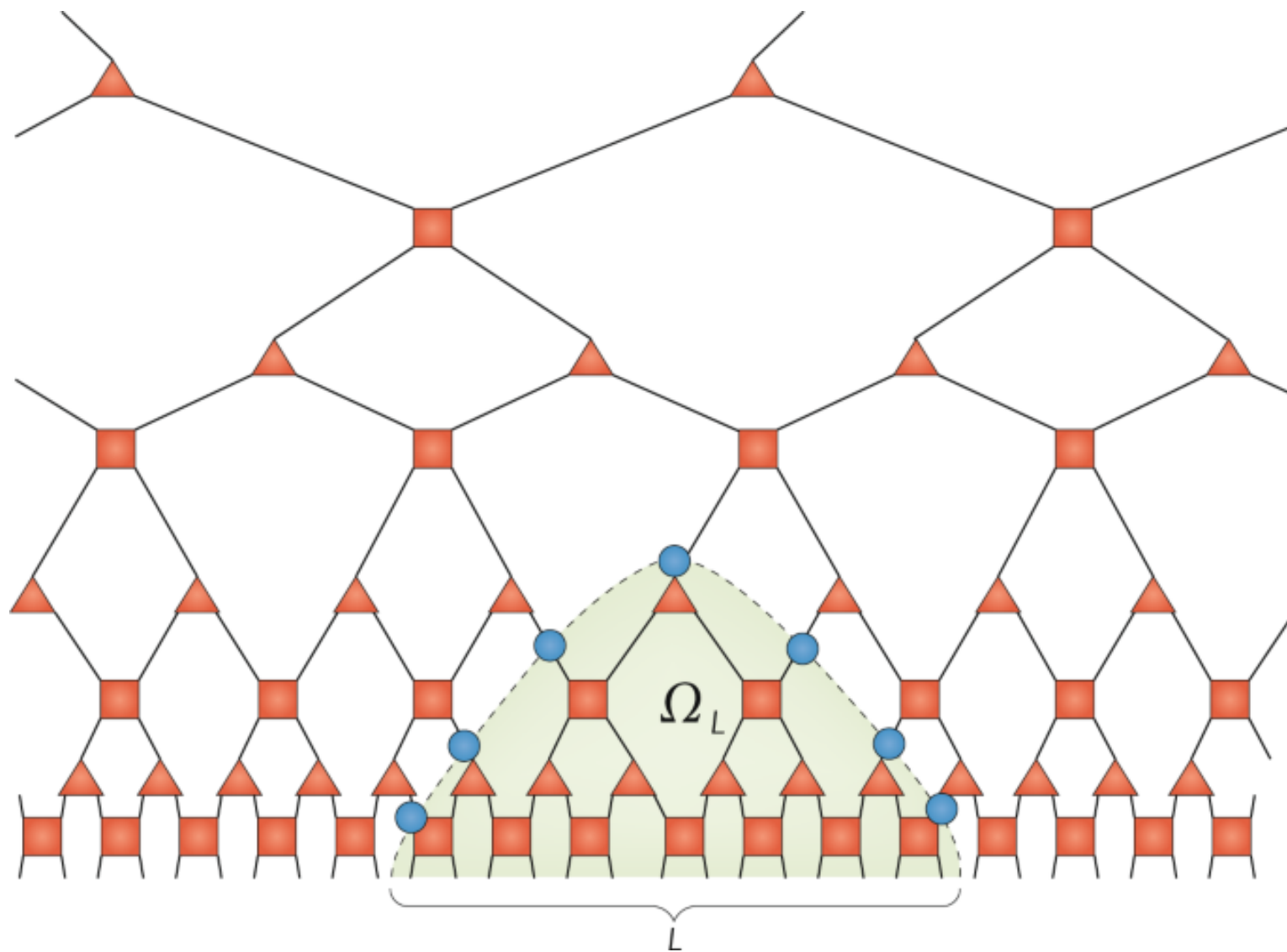
<sup>1</sup> ENS de Lyon, Université de Lyon, CNRS, INRIA, France

<sup>2</sup> University of Southampton, UK

**Abstract.** Bialgebras and Frobenius algebras are different ways in which monoids and comonoids interact as part of the same theory. Such theories feature in many fields: e.g. quantum computing, compositional semantics of concurrency, network algebra and component-based programming.

In this paper we study an important sub-theory of Coecke and Duncan’s ZX-calculus, related to strongly-complementary observables, where two Frobenius algebras interact. We characterize its free model as a category of  $\mathbb{Z}_2$ -vector subspaces. Moreover, we use the framework of PROPs to exhibit the modular structure of its algebra via a universal construction involving span and cospan categories of  $\mathbb{Z}_2$ -matrices and distributive laws between PROPs. Our approach demonstrates that the Frobenius structures result from the interaction of bialgebras.

non-example:



summary:

- Whitehead compositionality
- Schrödinger compositionality
- Complete compositionality

summary:

- Whitehead compositionality
- Schrödinger compositionality
- Complete compositionality
- LEGO compositionality





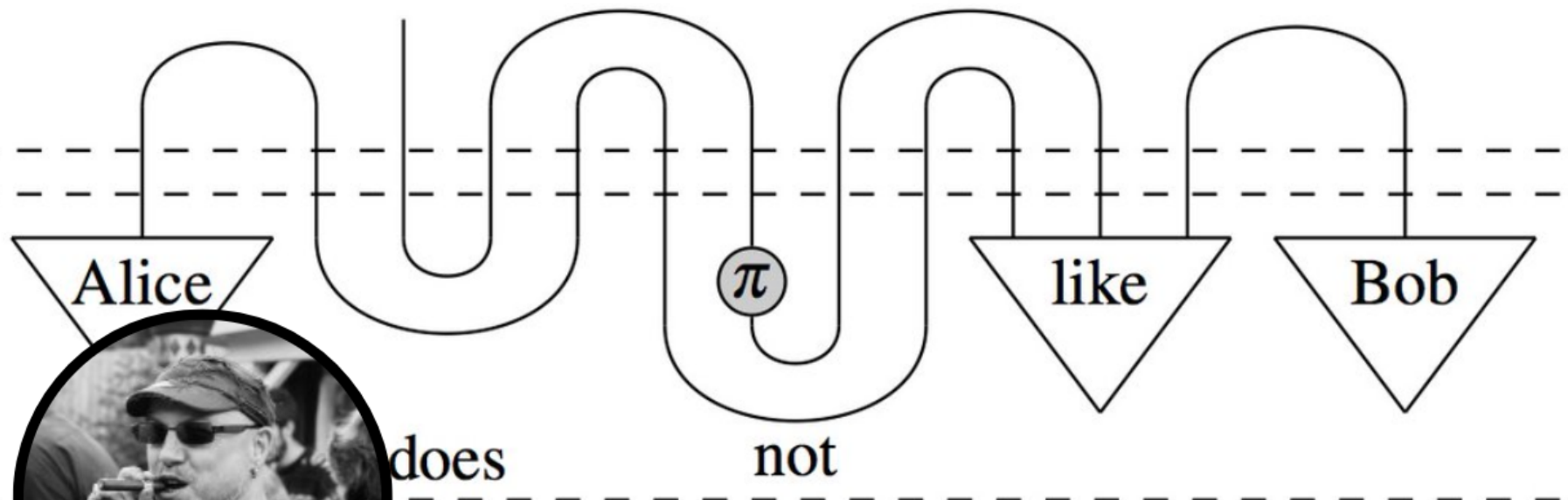
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@coecke

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Ex Professor @ Oxford University (CS)

Emeritus Professor @ Wolfson College (Ox)



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