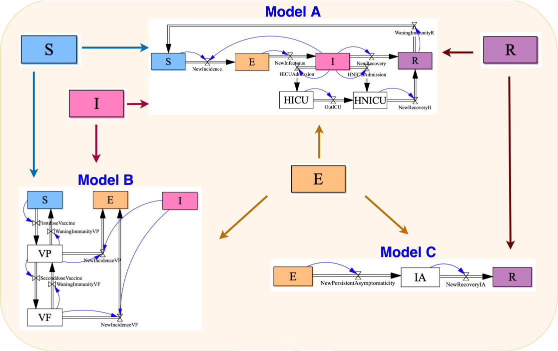
COMPOSITIONAL MODELING WITH STOCK-FLOW DIAGRAMS



John Baez, Xiaoyan Li, Nathaniel Osgood, Sophie Libkind and Evan Patterson

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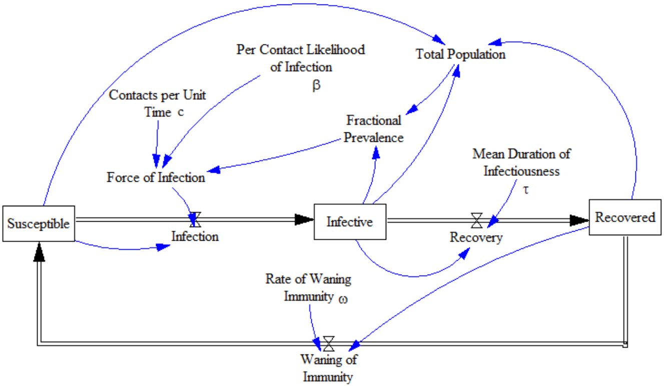
There is a community of epidemiologists who use “stock-flow diagrams” to model the spread of disease. This includes Nathaniel Osgood and Xiaoyan Li, who do COVID modeling for the Canadian government.



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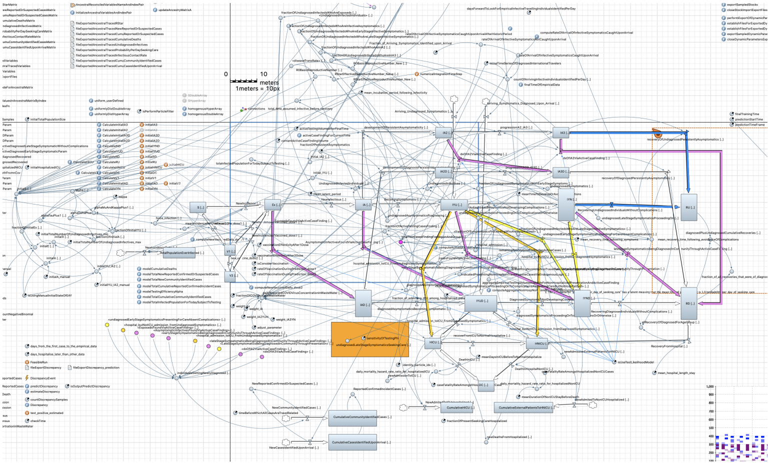
Unfortunately, modeling with stock-flow diagrams is done using software that does not support compositional modeling — most commonly AnyLogic.

Here is Osgood and Li's COVID model used by the government of Canada, in AnyLogic:



Applied category theory to the rescue!

Together with Evan Patterson, Sophie Libkin and myself, Osgood and Li have now created StockFlow: software that supports compositional modeling with stock-flow diagrams.

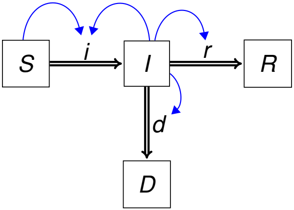
We used AlgebraicJulia: a framework for high-performance scientific computing that lets you program using category theory. This is being developed by a team including James Fairbanks, Evan Patterson, Sophie Libkind and many others.

Let me sketch the math underlying our software. I'll only do a simplified version of stock-flow diagrams.

In its simplest form, a **stock-flow diagram** consists of finite sets and functions:

$$
\begin{tikzcd}
\text{Flows} \arrow[rr, shift left, "u"] \arrow[rr, shift right, "d"'] & & \text{Stocks} \\
& \text{Links} \arrow[ul, "t"] \arrow[ur, "s"'] &
\end{tikzcd}
$$

together with, for each $f \in \text{Flows}$, a function $\phi_f \colon \mathbb{R}^{L(f)} \to \mathbb{R}$ where $L(f)$ is the set of links whose target is f.

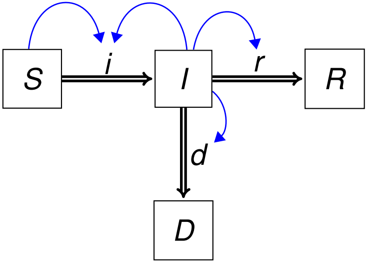


Define a **dynamical system** on a finite set N to be a continuous vector field v on $\mathbb{R}^N$. This gives a differential equation

$$\frac{d}{dt}x(t) = v(x(t))$$

describing how the stocks $x(t) \in \mathbb{R}^N$ change with time.

Each stock-flow diagram with set N of stocks gives a dynamical system on N:



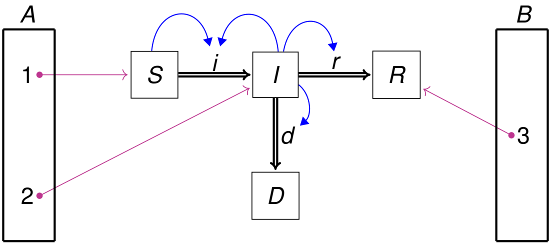
$$\frac{dS}{dt} = -\phi_i(S, I)$$

$$\frac{dI}{dt} = \phi_i(S, I) - \phi_r(I) - \phi_d(I)$$

$$\frac{dR}{dt} = \phi_r(I)$$

$$\frac{dD}{dt} = \phi_d(I)$$

Using the theory of decorated cospans, there is a category Open(StockFlow) where objects are finite sets and morphisms are *open* stock-flow diagrams:



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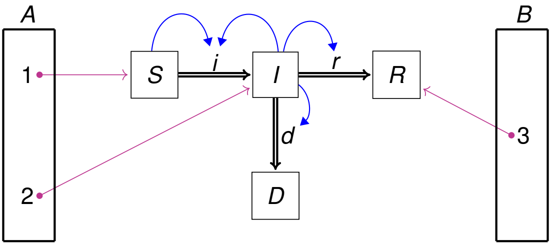


(Well, really *isomorphism classes* of open stock-flow diagrams.)

There is also a category Open(Dynam) where objects are finite sets and morphisms are open dynamical systems:

$$A \xrightarrow{i} N \xleftarrow{o} B \qquad\qquad v \in \mathrm{Vect}(N)$$

where $\mathrm{Vect}(N)$ is the set of continuous vector fields on $\mathbb{R}^N$.

(Again, we really need isomorphism classes of open dynamical systems.)

There is a functor

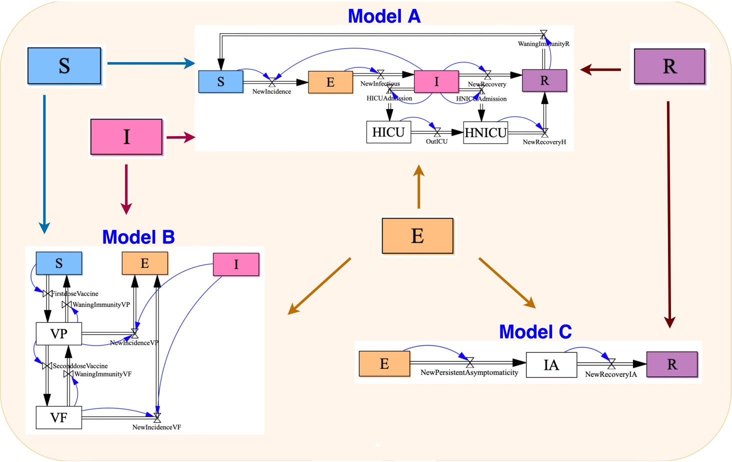
$$v\colon \mathrm{Open}(\mathrm{StockFlow}) \to \mathrm{Open}(\mathrm{Dynam})$$

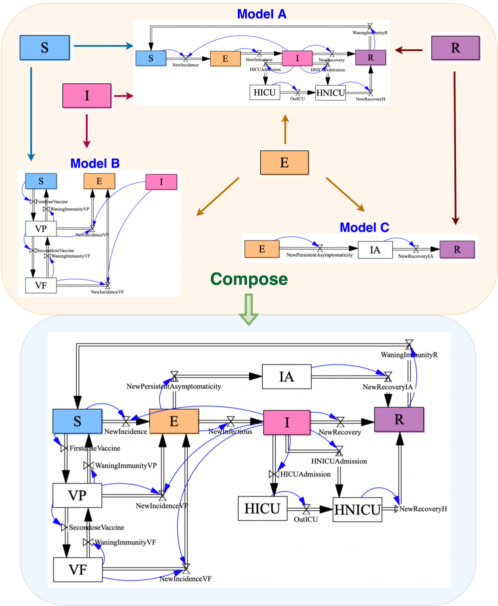
sending each finite set to itself and each open stock-flow diagram to the corresponding open dynamical system.

By implementing some of these ideas in AlgebraicJulia, my coauthors created a software package called **StockFlow**, now available on GitHub. This lets you:

- compose open stock-flow diagrams
- turn open stock-flow diagrams into open dynamical systems
- *solve* the differential equations given by these dynamical systems.

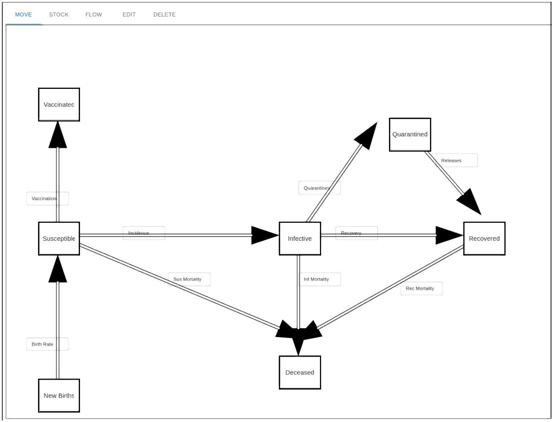
StockFlow lets you compose not only decorated cospans, but also “multicospans”:





Model A
S
I
R
E
HICU
HNICU
NewIncidence
NewInfectious
NewRecovery
HICUAdmission
HNICUAdmission
OutICU
NewRecoveryH
WaningImmunityR
Model B
VP
VF
FirstdoseVaccine
WaningImmunityVP
NewIncidenceVP
SeconddoseVaccine
WaningImmunityVF
NewIncidenceVF
Model C
IA
NewPersistentAsymptomaticity
NewRecoveryIA
Compose

Osgood, Li and others are building a graphical interface for StockFlow. This allows *teams* to *collaboratively* build stock-flow models on their web browsers. Existing software is single-user, not web-based, and does not allow composition.



In August 2022, Osgood, Libkind, Patterson, Li, Fairbanks and others will teach a week-long course on StockFlow. This is being funded by the Canadian Network for Modelling Infectious Disease (CANMOD).

So, applied category theory can be a practical tool!

Here's our paper:

- Compositional modeling with stock and flow diagrams, arXiv:2205.08373.