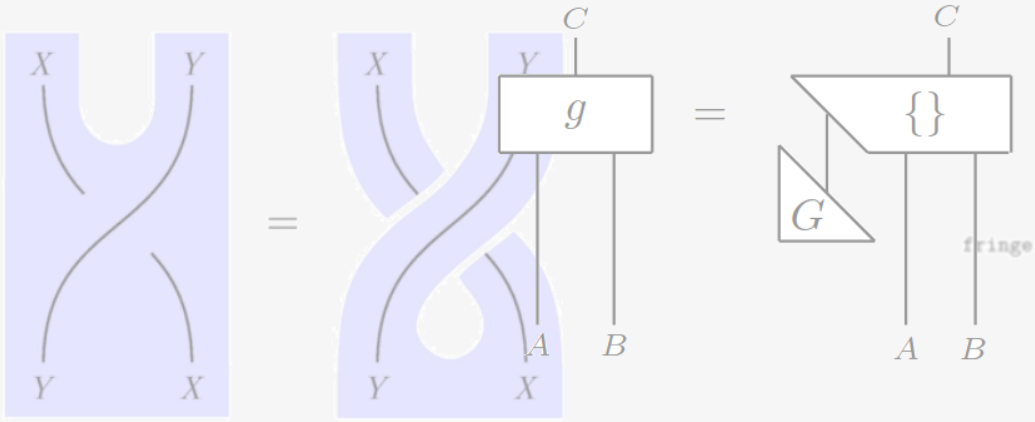
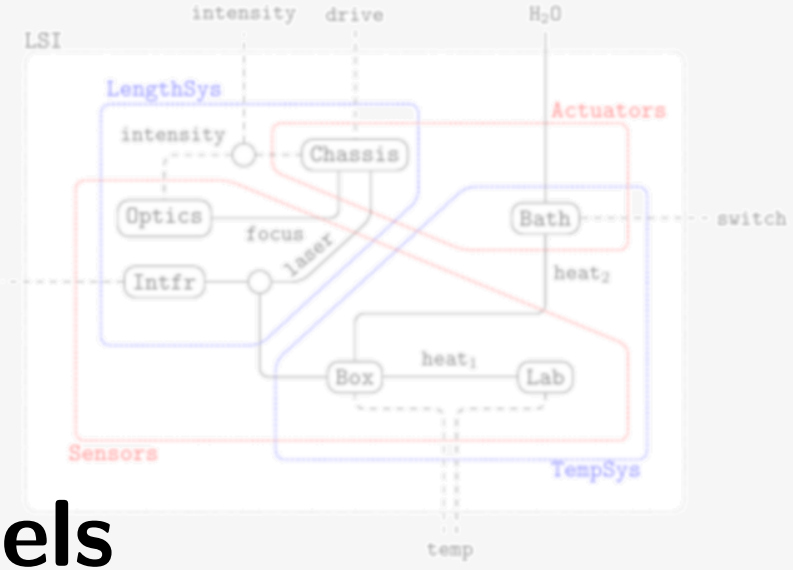
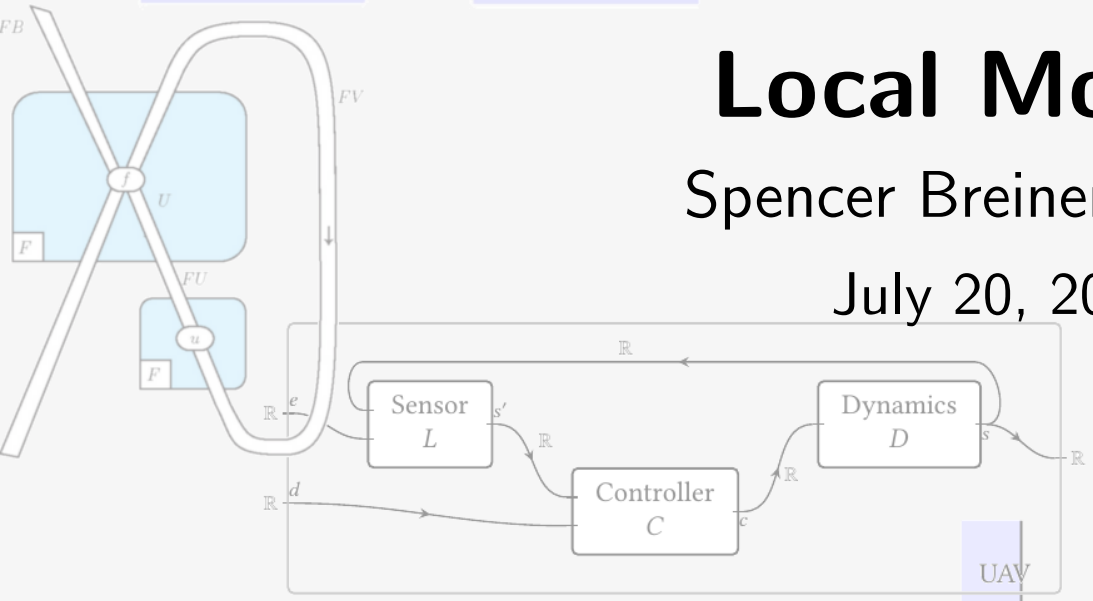


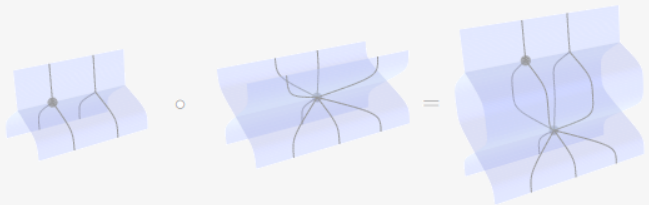


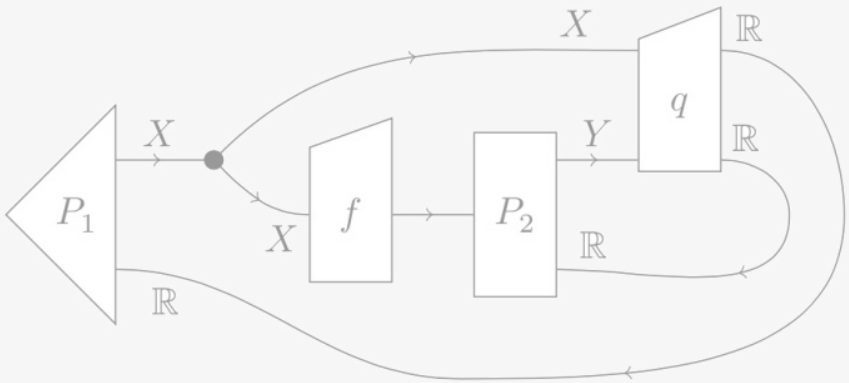
Local Models

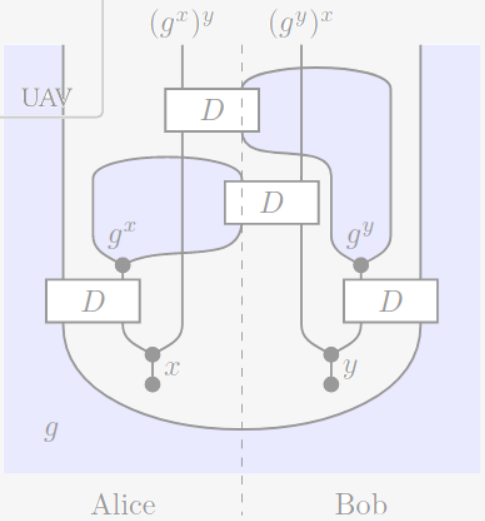
Spencer Breiner - NIST

July 20, 2022









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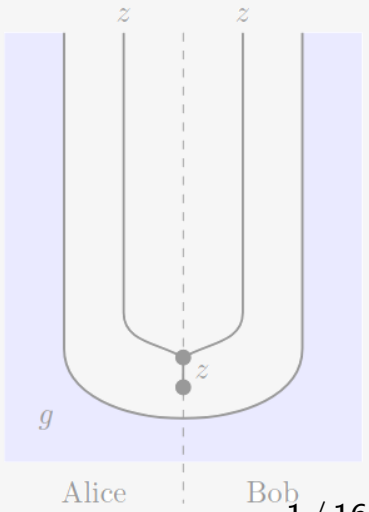
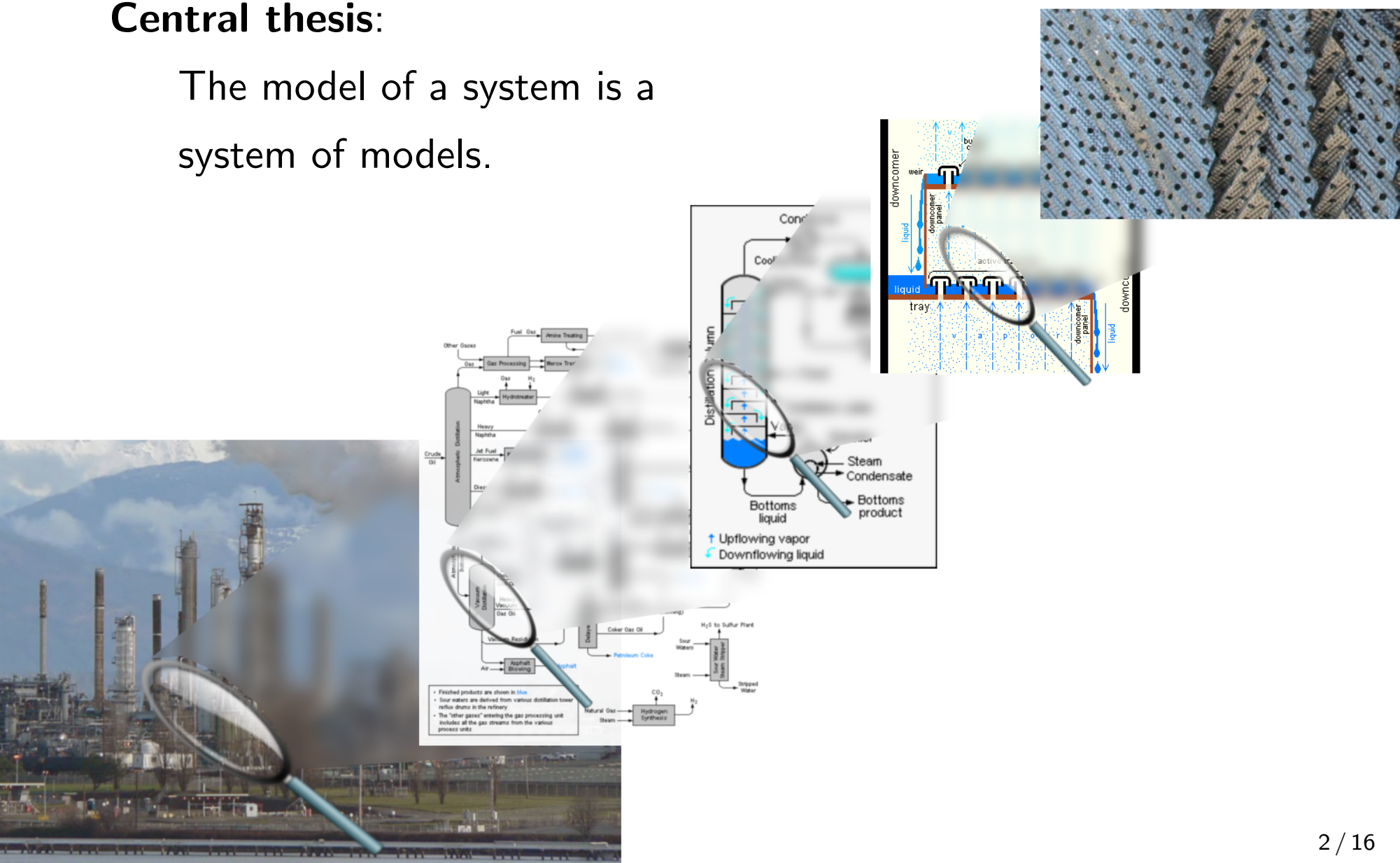
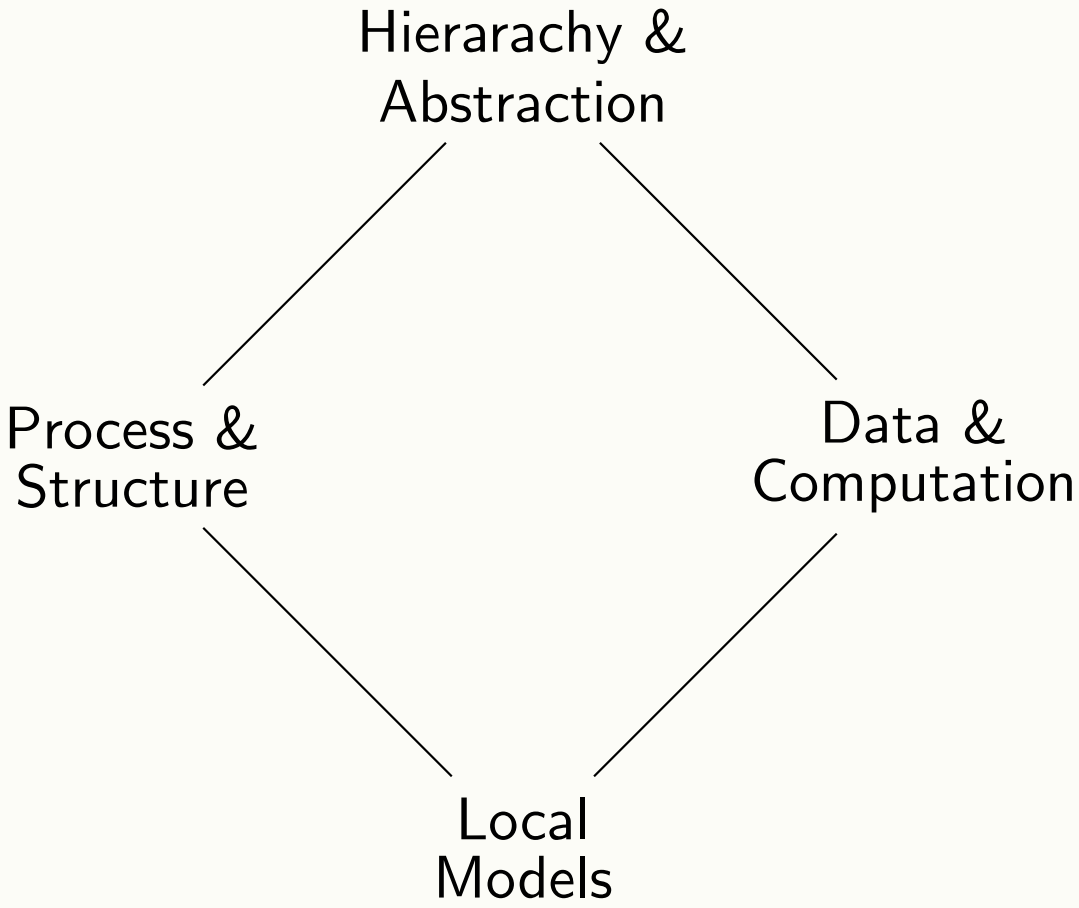


Central thesis:

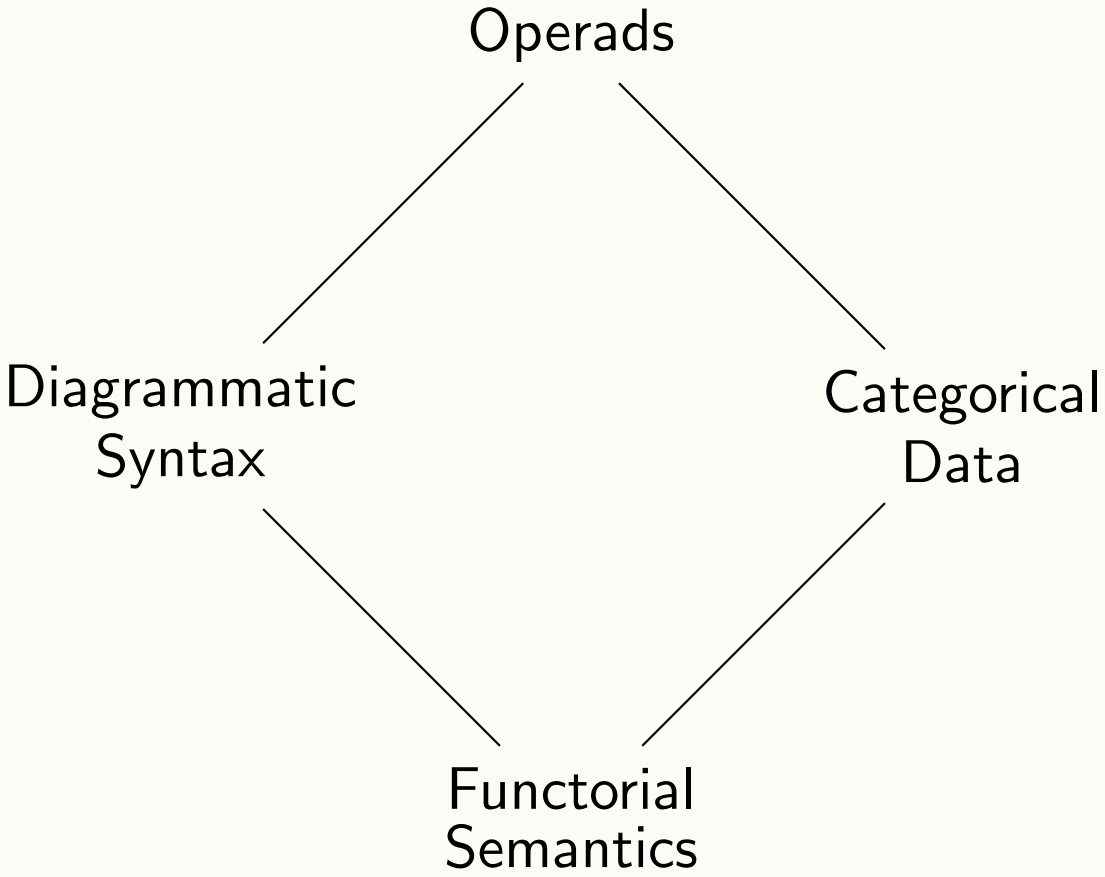
The model of a system is a system of models.



Framing systems



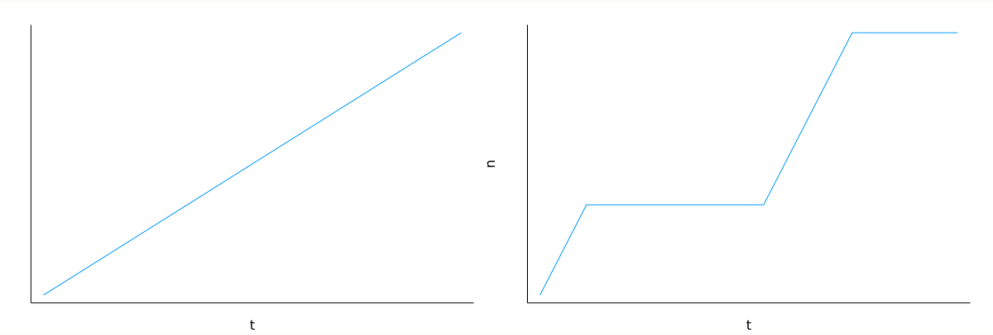
Framing systems



Local Models

$$f_1(x : \mathbb{R}) = (x + .1, \cos(x) > .5)$$
$$f_2(b : \mathbb{B}, n : \mathbb{N}) = \text{if } b \text{ then } n + 1 \text{ else } n$$
$$\hline$$
$$f : \mathbb{R} \times \mathbb{N} \longrightarrow \mathbb{R} \times \mathbb{N} \in \mathsf{Set}$$

$$s_0, f(s_0), f(f(s_0)), \ldots, f^k(s_0), \ldots$$



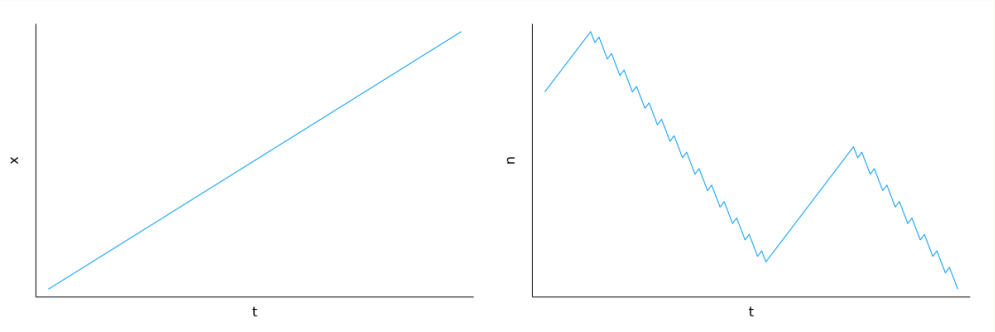
Local Models

$$f_1(x : \mathbb{R}) = (x + .1, \cos(x) > .5)$$

$$f_2'(b, n) = \text{if } b \text{ or } n \equiv 0 \pmod 3 \text{ then } n + 1 \text{ else } n - 2$$

$$f' : \mathbb{R} \times \mathbb{N} \longrightarrow \mathbb{R} \times \mathbb{N} \in \mathsf{Set}$$

$$s_0, f'(s_0), f'(f'(s_0)), \ldots, f'^k(s_0), \ldots$$



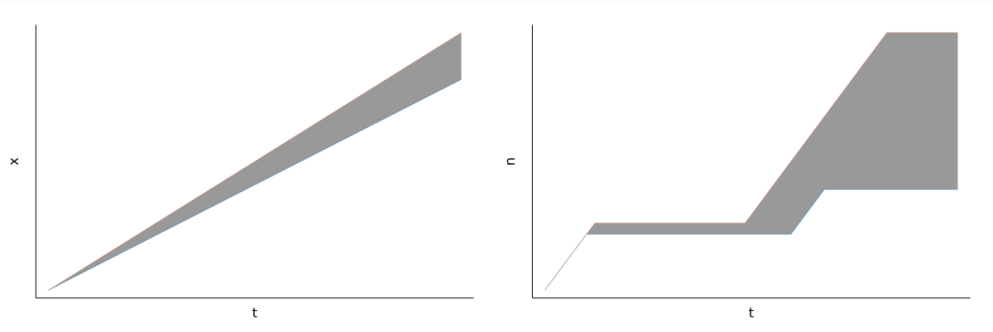
Local Models

$$R_1(x : \mathbb{R}) = (x + .1 \pm .01, \cos(x_1) > .5)$$

$$f_2(b, n_1) = \text{if } b \text{ then } n_1 + 1 \text{ else } n$$

$$R : \mathbb{R} \times \mathbb{N} \multimap \mathbb{R} \times \mathbb{N} \in \mathsf{Rel}$$

$$s_0, R(s_0), R(R(s_0)), \ldots, R^k(s_0), \ldots$$



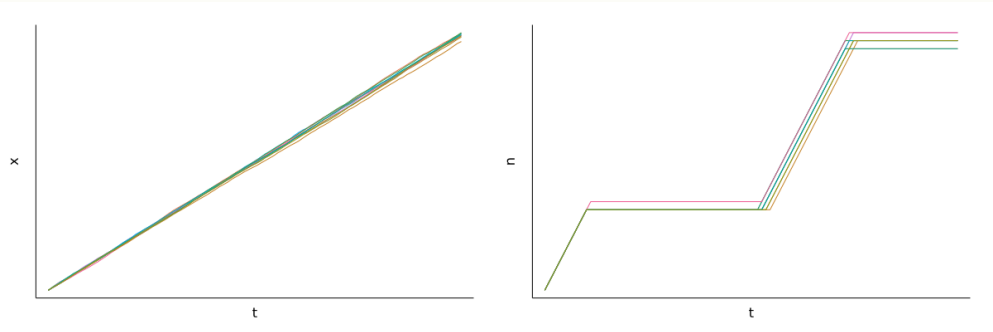
Local Models

$$p_1(x : \mathbb{R}) = (x + .1 + N(0, .01), \cos(x_1) > .5)$$

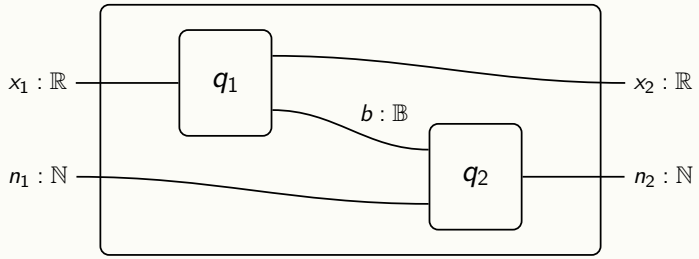
$$f_2(b, n_1) = \text{if } b \text{ then } n_1 + 1 \text{ else } n$$

$$p : \mathbb{R} \times \mathbb{N} \rightsquigarrow \mathbb{R} \times \mathbb{N} \in \text{Prob}$$

$$s_0, p(s_0), p(p(s_0)), \ldots, p^k(s_0), \ldots$$

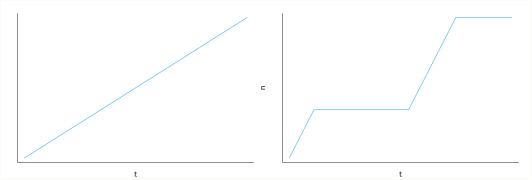


Process & Structure



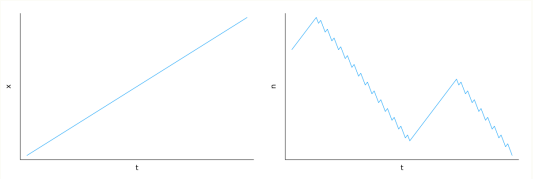
$$\mathsf{f} : \mathcal{P} \longrightarrow \mathsf{Set}$$

$$q_1 \mapsto f_1, q_2 \mapsto f_2$$



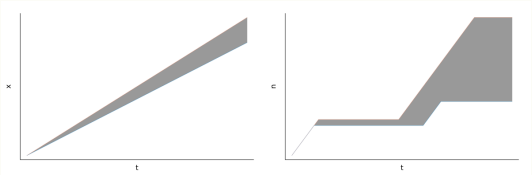
$$\mathsf{f}' : \mathcal{P} \longrightarrow \mathsf{Set}$$

$$q_1 \mapsto f_1, q_2 \mapsto f_2'$$



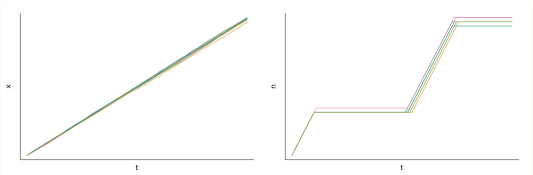
$$\mathsf{R} : \mathcal{P} \longrightarrow \mathsf{Rel}$$

$$q_1 \mapsto R_1, q_2 \mapsto \eta_{\mathcal{P}}(f_2)$$

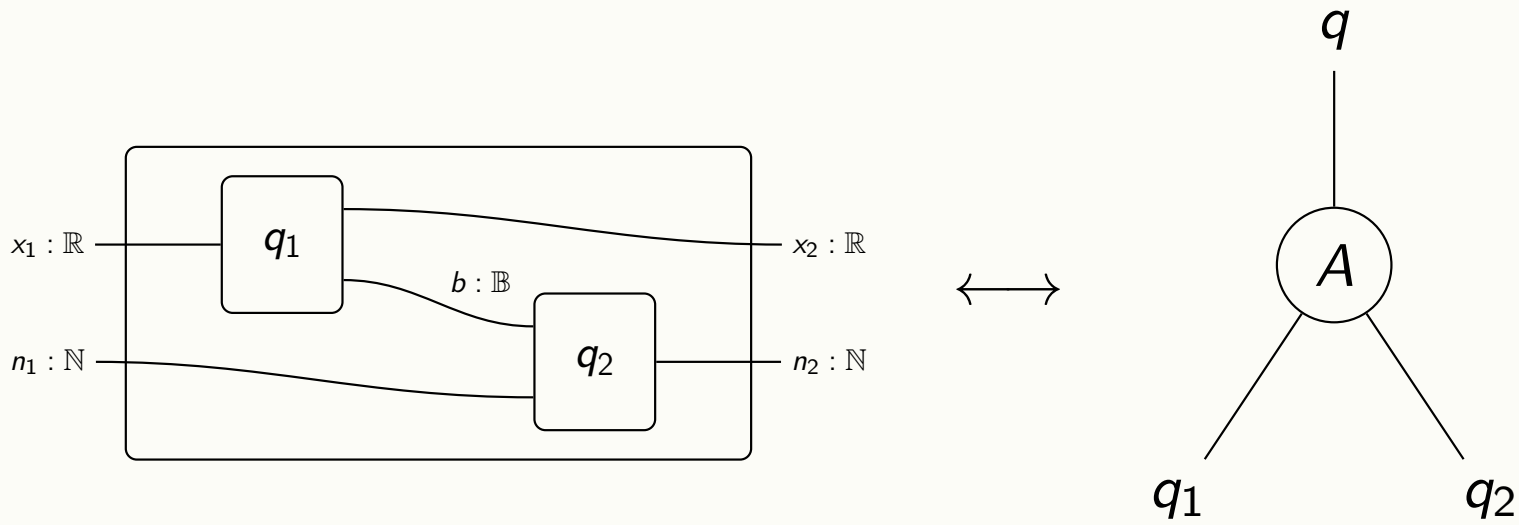


$$\mathsf{p} : \mathcal{P} \longrightarrow \mathsf{Prob}$$

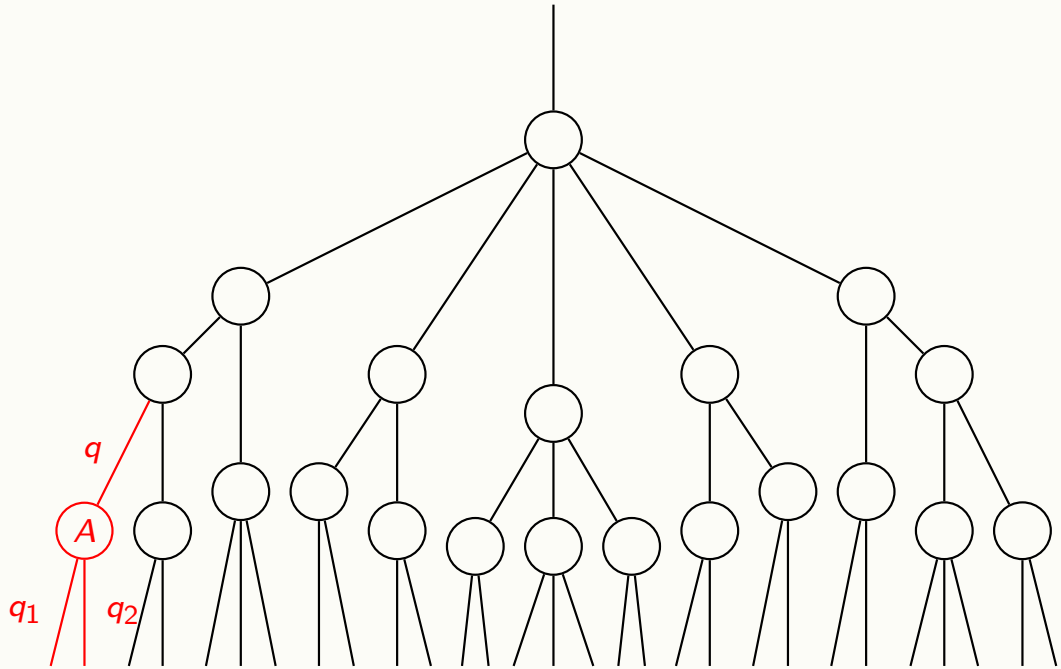
$$q_1 \mapsto p_1, q_2 \mapsto \eta_{\mathcal{D}}(f_2)$$



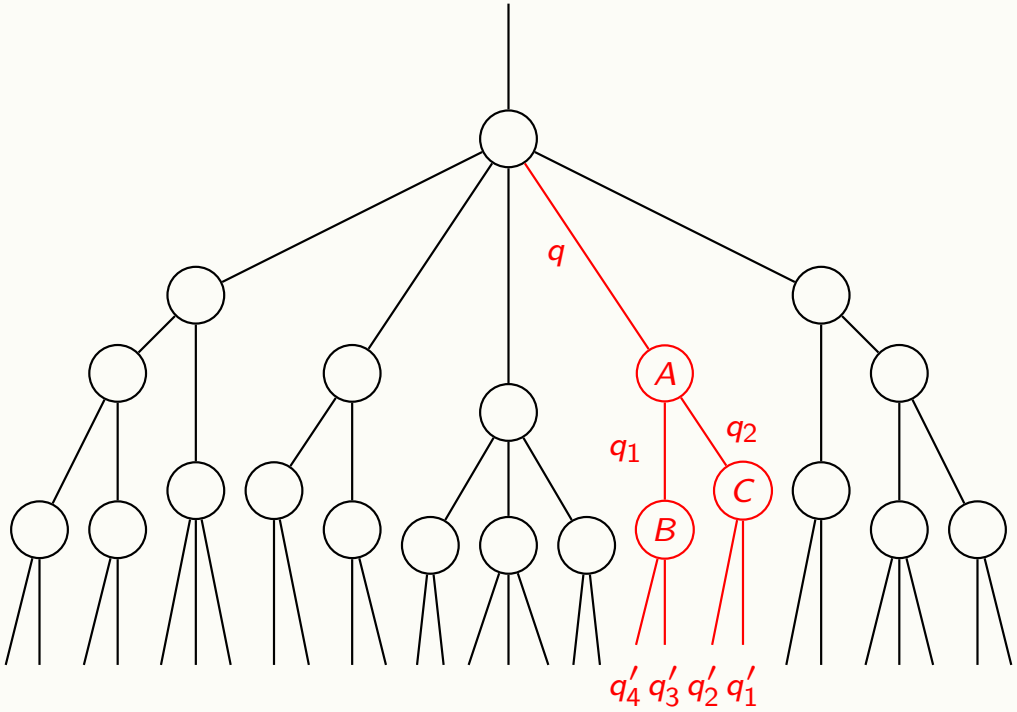
Hierarchy & Abstraction



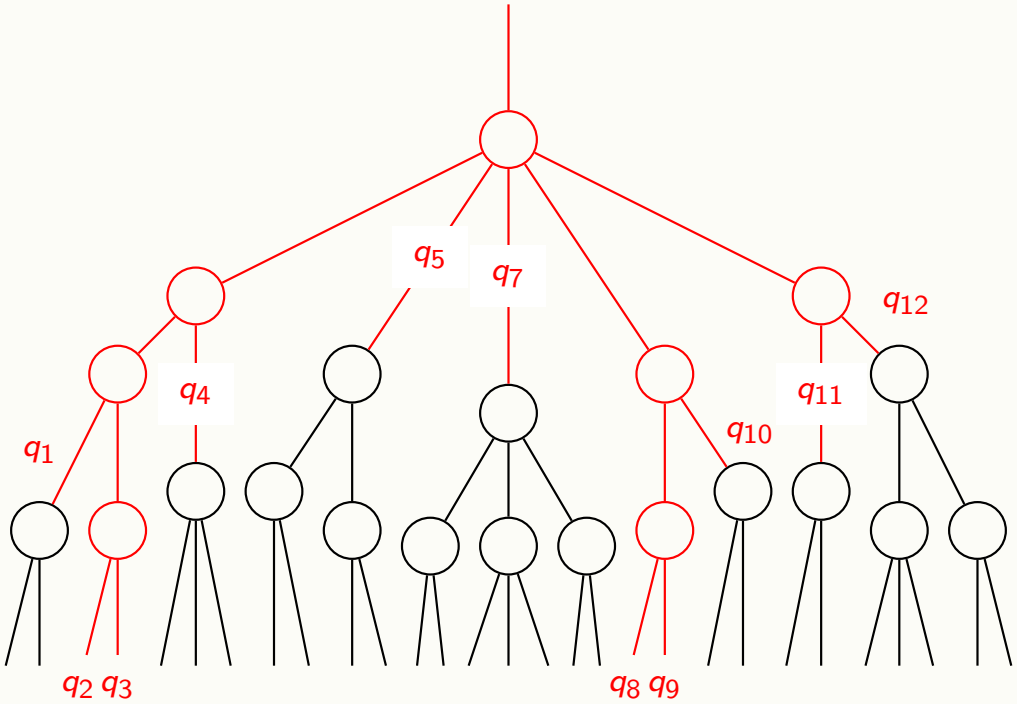
Hierarchy & Abstraction



Hierarchy & Abstraction



Hierarchy & Abstraction



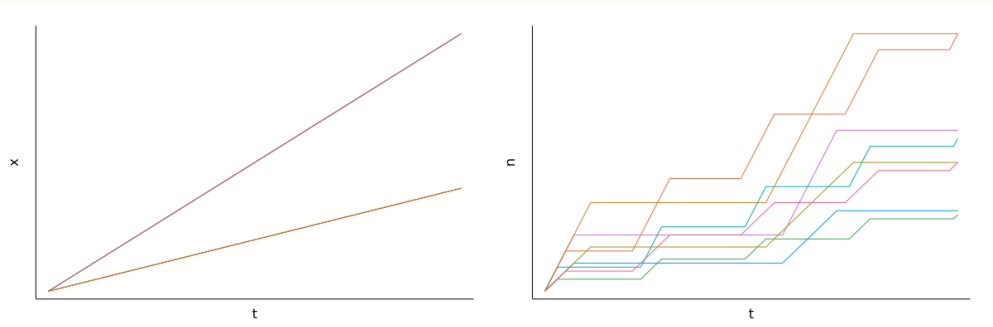
Data & Computation

$$f_1(x_1) = (x_1 + q_1.\delta, \cos(x_1) > q_1.\ell)$$

$$f_2(b, n_1) = \text{if } b \text{ then } n_1 + q_2.\kappa \text{ else } n_1$$

q_1	δ	ℓ
1	.1	.5
2	.1	.8
3	.25	.5
4	.25	.8

q_2	κ	μ
1	1	.1
2	2	.01



Systems models

Definition (provisional)

- An *abstraction operad* $\mathcal{A}$ (often free).
- A *diagram functor* $\mathcal{D} : \mathcal{A} \to \mathbf{Diag}$
- A *query functor* $\mathcal{Q} : \mathcal{A} \to \mathbf{CData}_*$
- A set of *local models* $L \in \mathcal{L}$ with
  - A *context assignment* $a_L : \langle p_i \rangle \to p \in \mathcal{A}$
  - A *semantic category* $\mathcal{S}_L$
  - A *semantic functor* $\mathrm{Set}^{\mathcal{Q}(a_L)} \times \mathcal{D}(a_L) \to S_L$