Temporal Explorability Games

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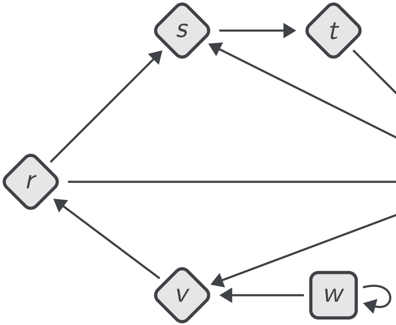
Reachability Games

Example

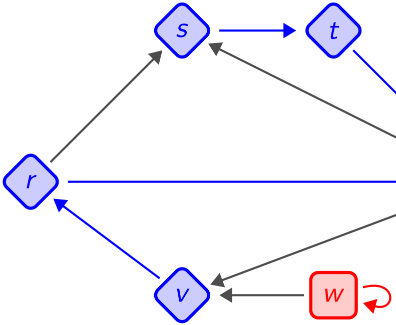
Reach $= \Diamond$, Safety $= \Box$



Reachability Games

Example

Reach $= \Diamond$, Safety $= \Box$, Reach Player Win/Strategy, Safety Player Win/Strategy

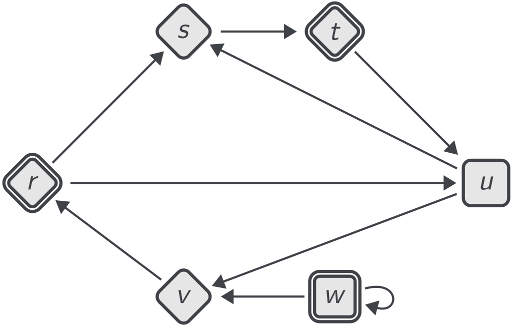


Generalised Reachability Games

Example

Objective: Reach at least one vertex from ALL target subsets

Reach $= \Diamond$, Safety $= \Box$, Target $= \{\{r, w\}, \{t, w\}\}$

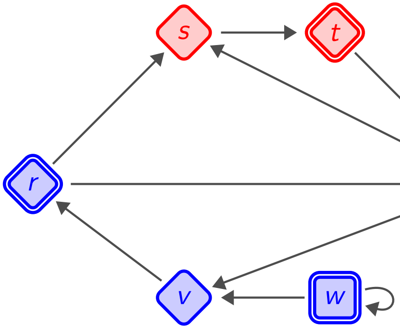


Generalised Reachability Games

Example

Target $= \{\{r, w\}, \{t, w\}\}$

Reach $= \diamondsuit$, Safety $= \square$, Reach Player Win, Red $=$ Safety Player Win



Generalised Reachability Games

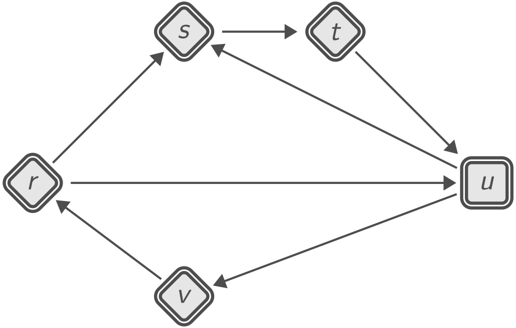
- Fijalkow and Horn found that solving Generalised Reachability Games is PSPACE-Complete (assuming target subset size is ≥ 3)
- When target subsets are singletons, they found solving these games is P-Complete
- Explorability is when every vertex is a target singleton

Explorability Games

Example

Target $= \{\{r\}, \{s\}, \{t\}, \{u\}, \{v\}\}$ i.e. Explorability

Reach $= \Diamond$, Safety $= \Box$

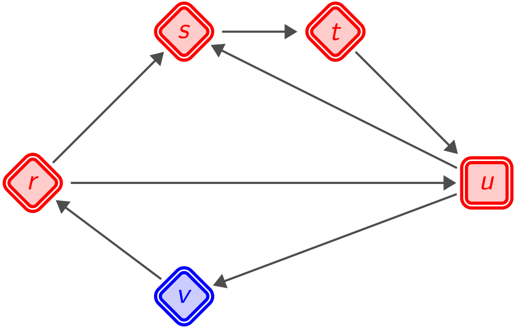


Explorability Games

Example

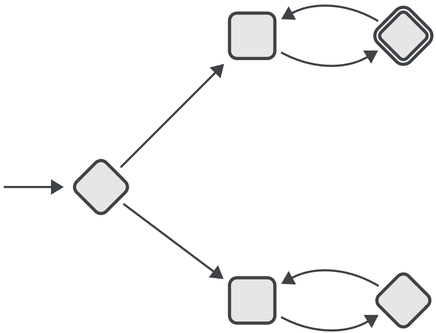
Target $= \{\{r\}, \{s\}, \{t\}, \{u\}, \{v\}\}$ i.e. Explorability

Reach $= \Diamond$, Safety $= \Box$, Reach Player Win, Red $=$ Safety Player Win



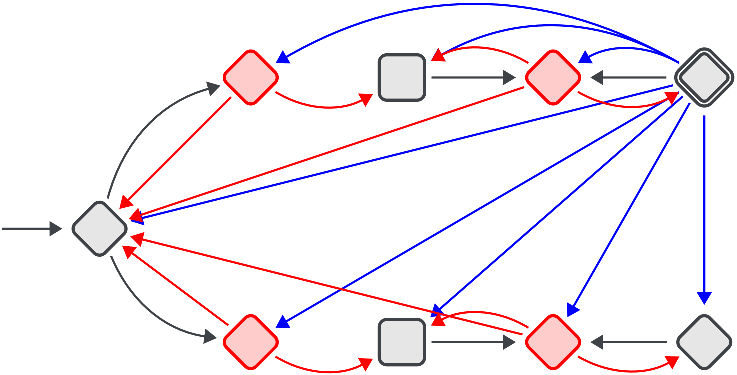
Explorability Games

Complexity



Explorability Games

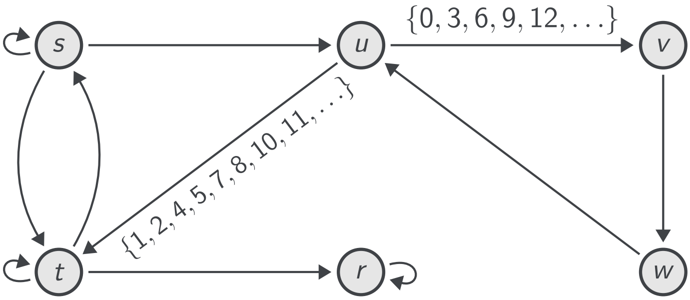
Complexity



Temporal Graphs

Example

Only a subset of edges available at a given timestep. In our model: No waiting allowed, time strictly increases for every edge taken

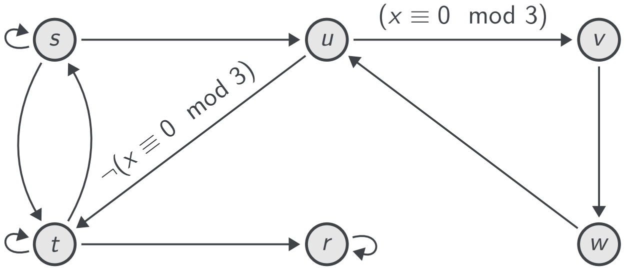


Temporal Graphs

Example

Existential Presburger Arithmetic ($\exists PA$)

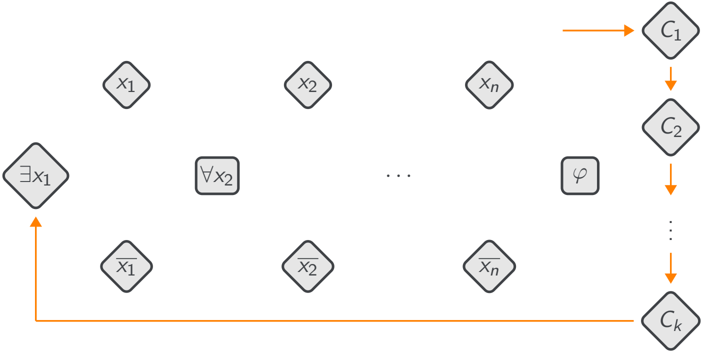
- $\exists PA$ fragment does not allow $\forall$, membership is NP-Complete
- $\exists PA$ can be used to encode temporal edges concisely



Explicit Temporal Explorability Games

Complexity

$$\varphi = \exists x_1 \forall x_2 \ldots \exists x_{n-1} \forall x_n ((x_1 \vee \overline{x_2} \vee x_n) \wedge \ldots \wedge (x_n \vee \overline{x_{n-1}} \vee \overline{x_1}))$$



Explicit Temporal Explorability Games

Complexity

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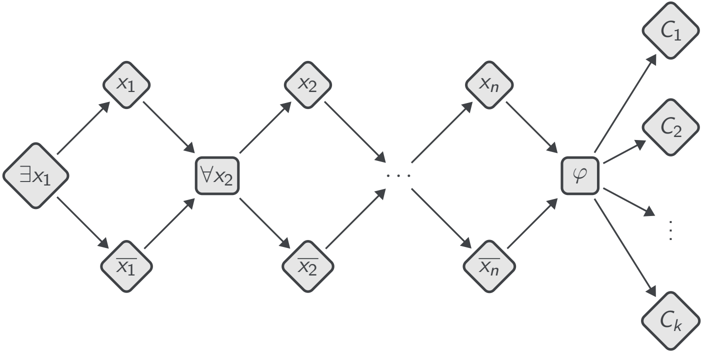

Explicit Temporal Explorability Games

Complexity

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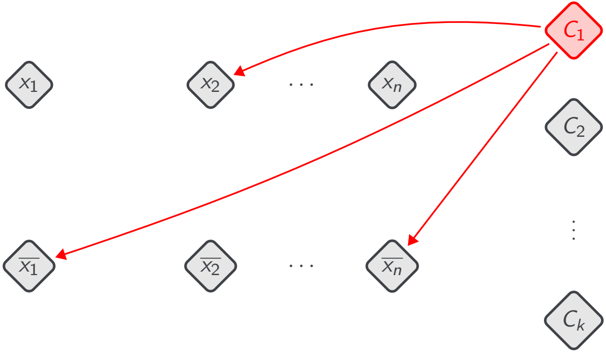
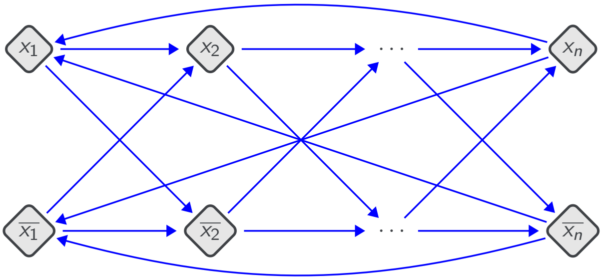


Explicit Temporal Explorability Games

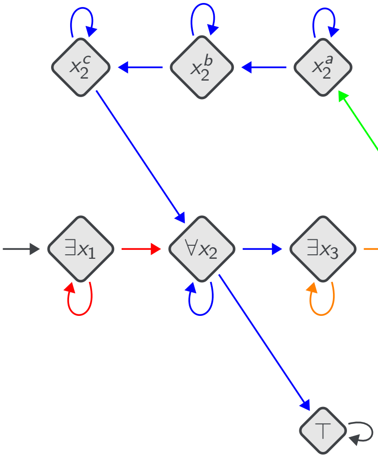
Complexity

$$\varphi = \exists x_1 \forall x_2 \ldots \exists x_{n-1} \forall x_n ((x_1 \vee \overline{x_2} \vee x_n) \wedge \ldots \wedge (x_n \vee \overline{x_{n-1}} \vee \overline{x}_1))$$



Symbolic Temporal Explorability Games

Complexity



Reachability Games

Generalised Reachability Games

Explorability Games

Temporal Graphs

Explicit Temporal Explorability Games

Symbolic Temporal Explorability Games

Summary

Summary

		Static	Explicit	Symbolic
One-player	Reachability	NL-complete [Arora and Barak, 2009]	NL-complete [Theorem 6]	PSPACE-complete [Corollary 14]
	Explorability	NL-complete [Theorem 5]	NP-complete [Theorem 7]	PSPACE-complete [Corollary 14]
	Gen. Reach	NP-complete [Fijalkow and Horn, 2012]	NP-complete [Theorem 6]	PSPACE-complete [Corollary 14]
Two-player	Reachability	P-complete [Grädel et al, 2002]	P-complete [Theorem 6]	PSPACE-complete [Austin et al, 2024]
	Explorability	P-complete [Theorem 5]	PSPACE-complete [Theorem 8]	PSPACE-hard; In EXP [Corollary 14]
	Gen. Reach	PSPACE-complete [Fijalkow and Horn, 2012]	PSPACE-complete [Theorem 6]	PSPACE-hard; In EXP [Corollary 14]

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