Ensuring Liveness Properties of Distributed Systems with Justness

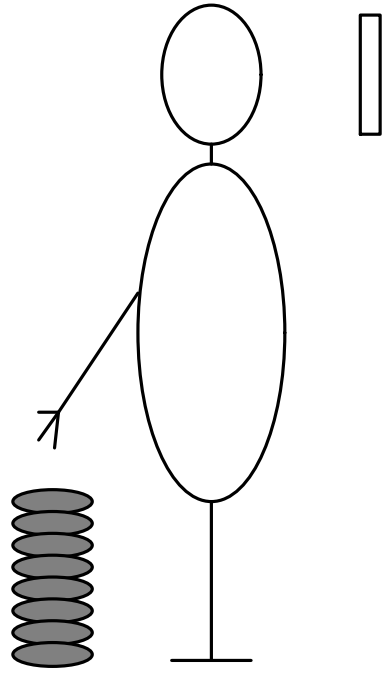
Rob van Glabbeek

University of Edinburgh

April 2025

Liveness properties – an example

Something good will eventually happen.



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Liveness property: at least 3 quarters will be inserted.

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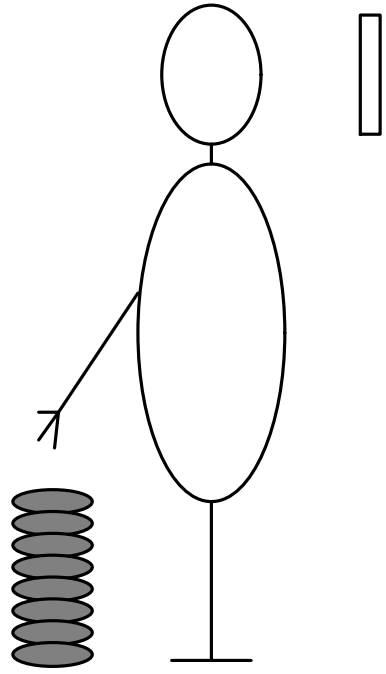

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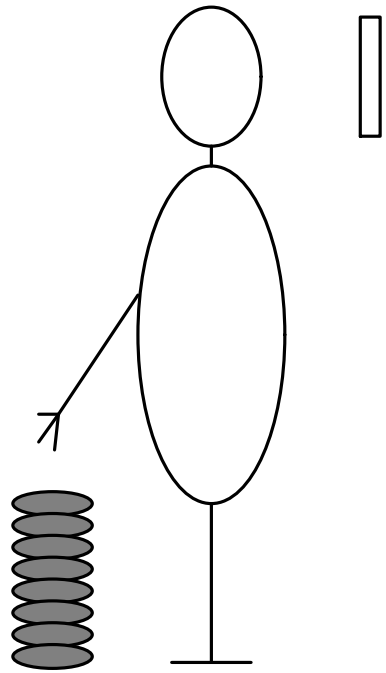

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The assumption that a system will not stop without a reason.

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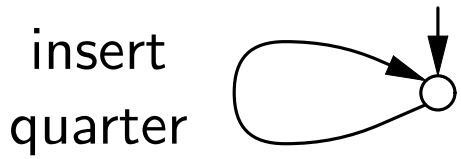
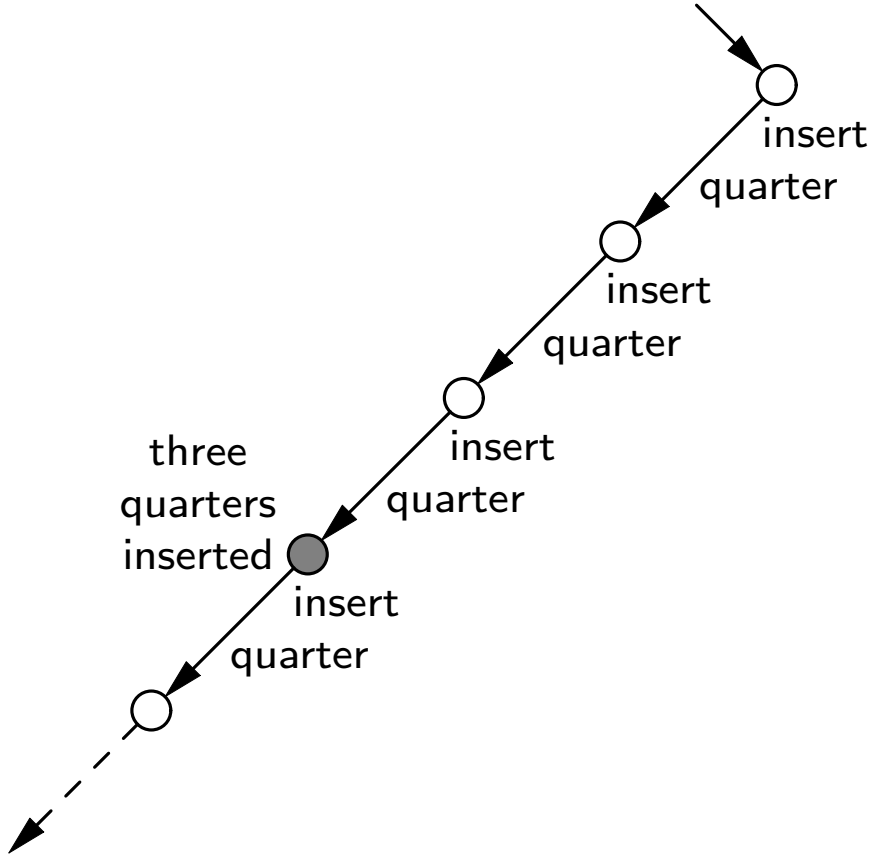


Transition system with success state



Progress, Justness, Fairness and Liveness

Fairness

$\Downarrow$

Justness

$\Downarrow$

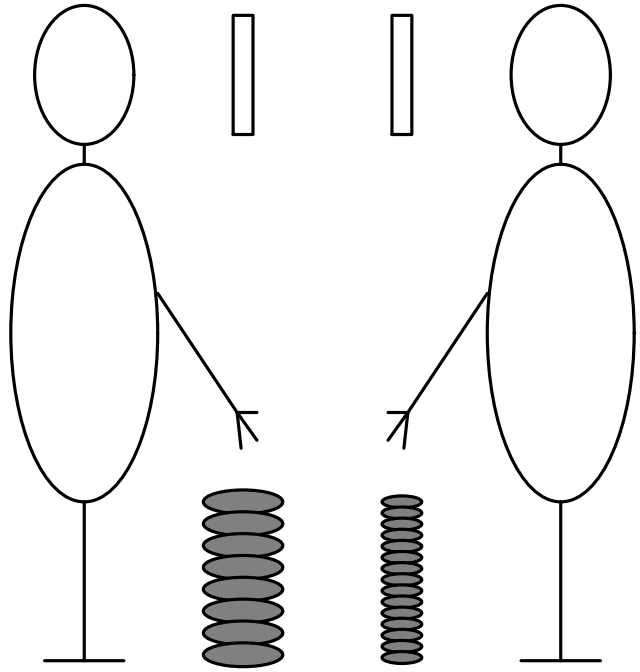
Progress

a hierarchy of assumptions

Liveness properties

somethings one want to obtain,
optionally
when making one such assumption

Liveness properties – a more interesting example



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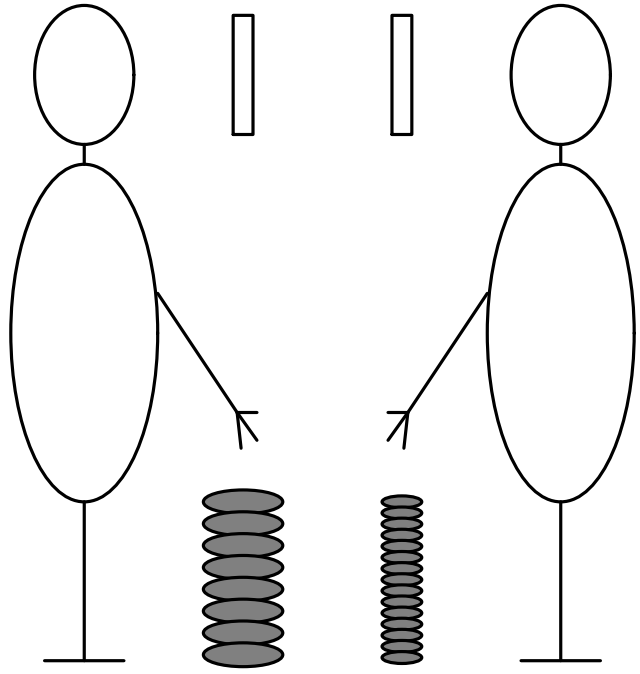

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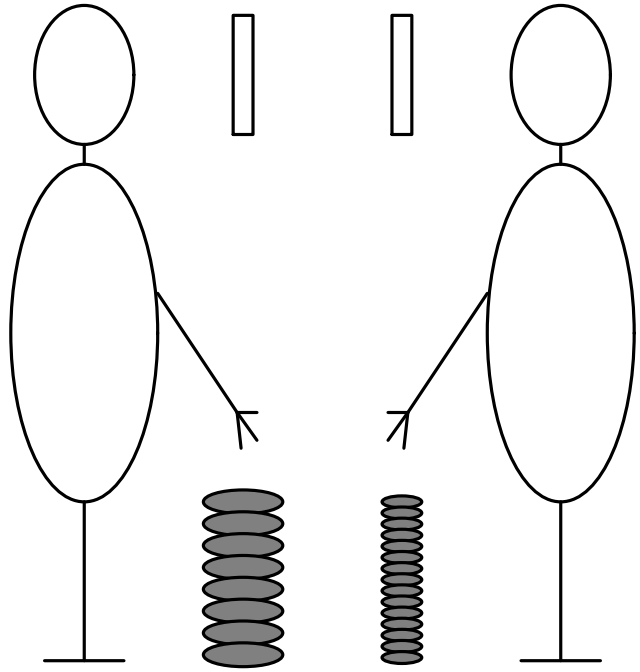

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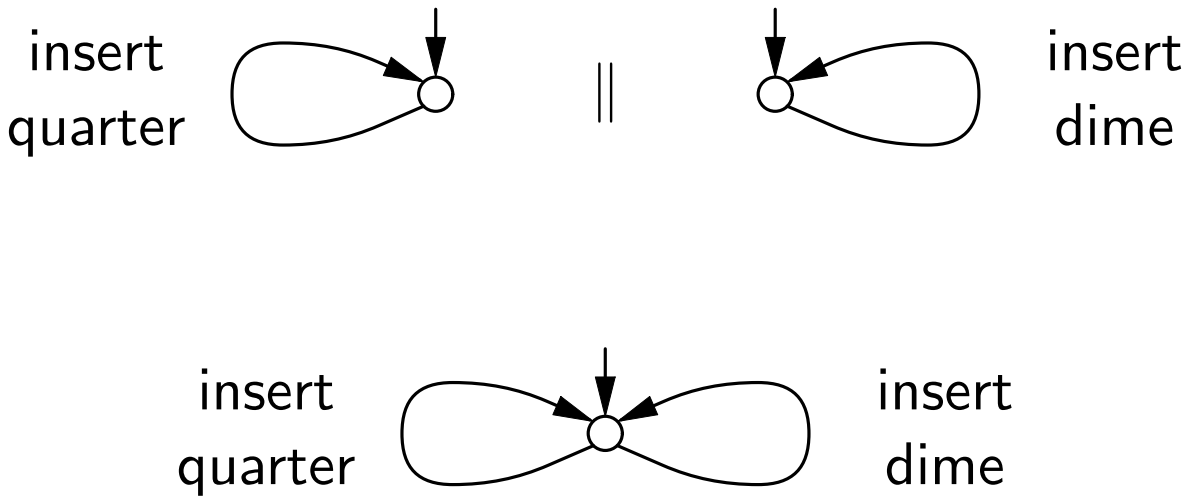

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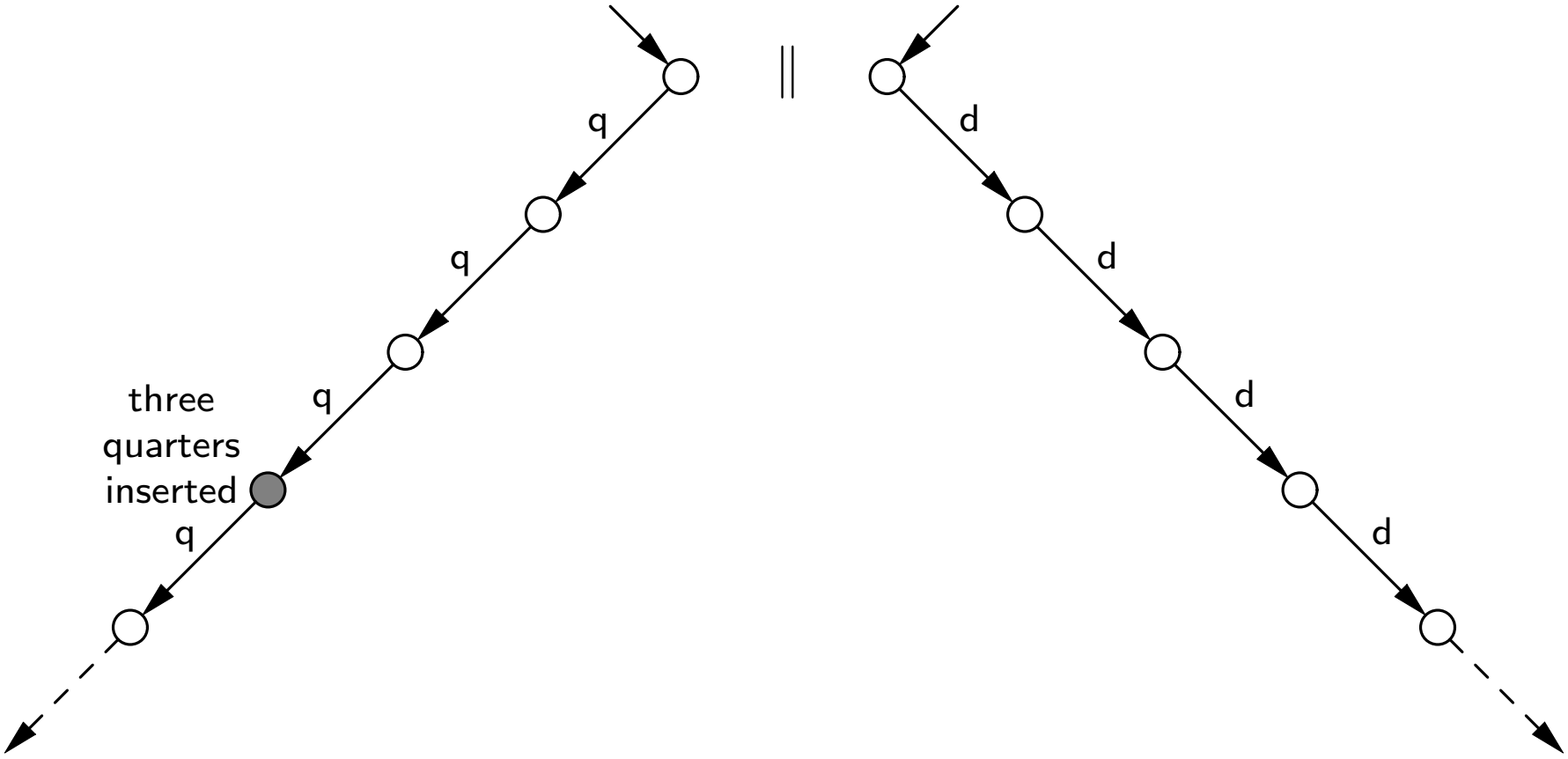

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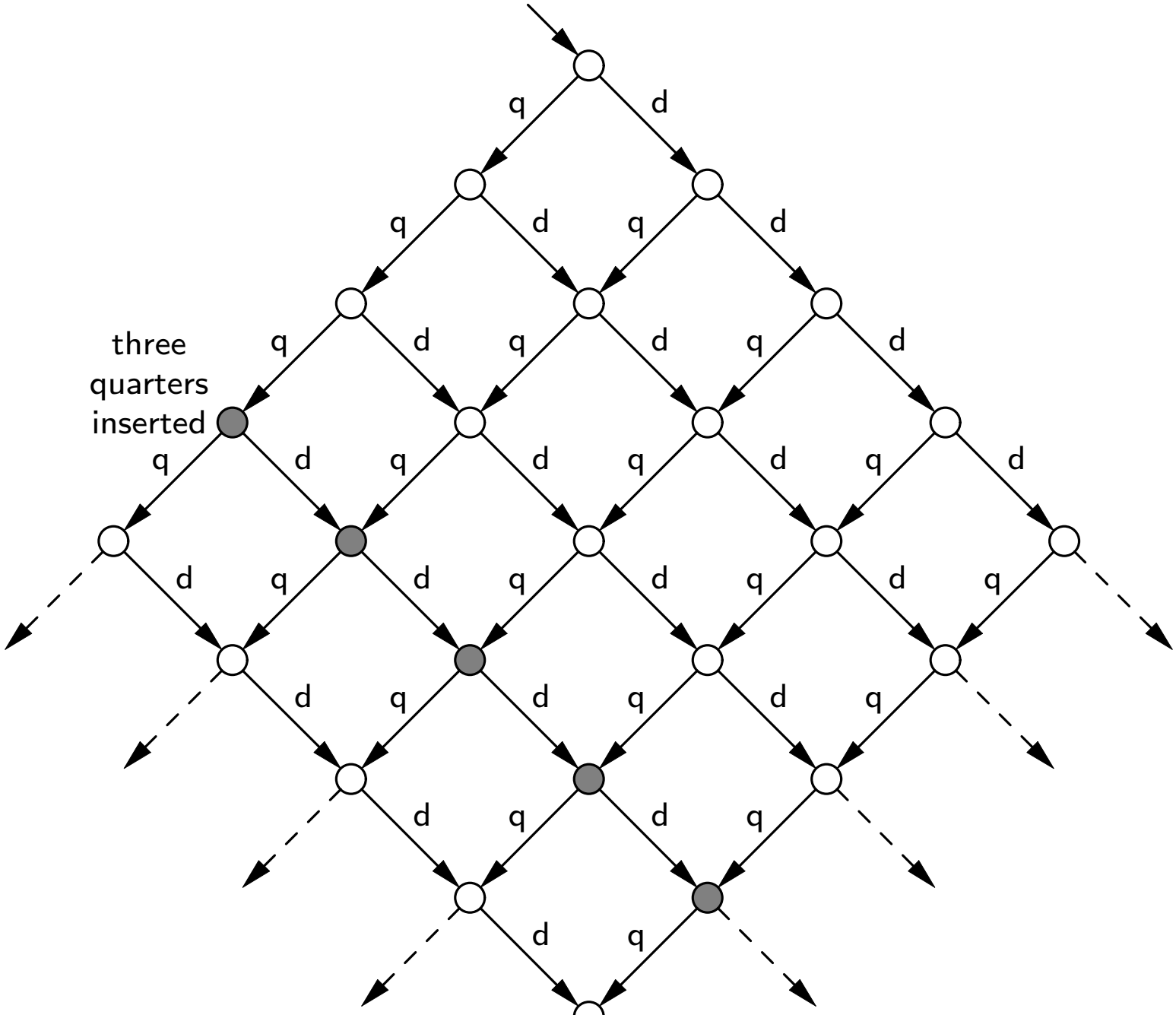
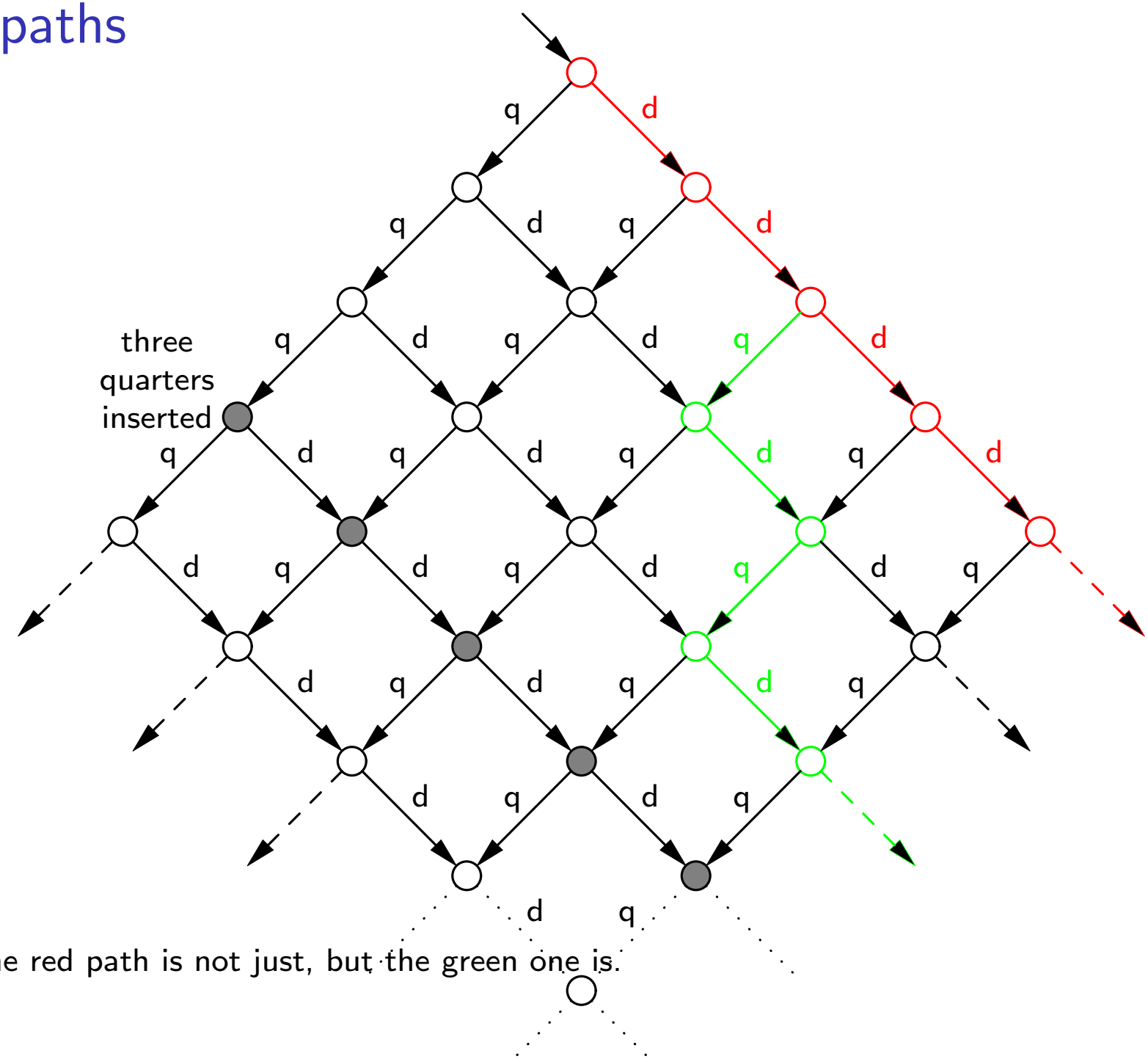


Just paths



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Concurrency versus competition

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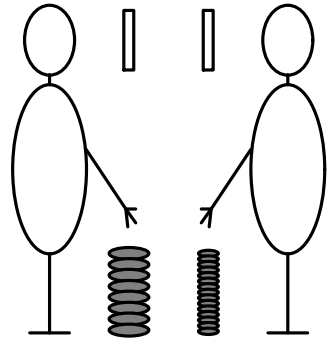

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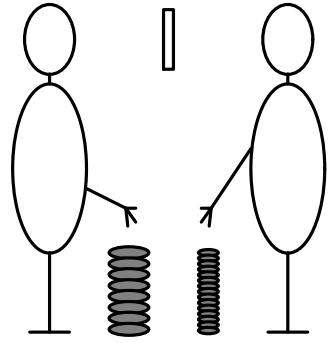

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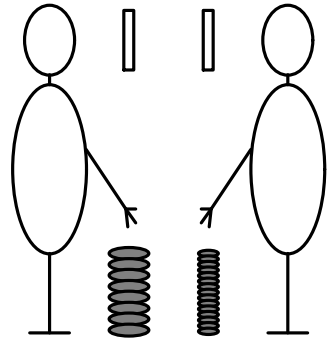

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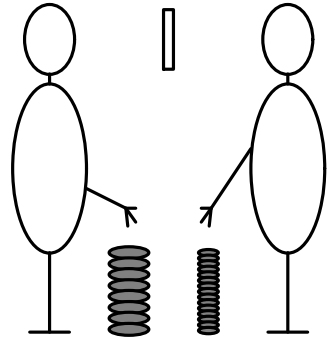

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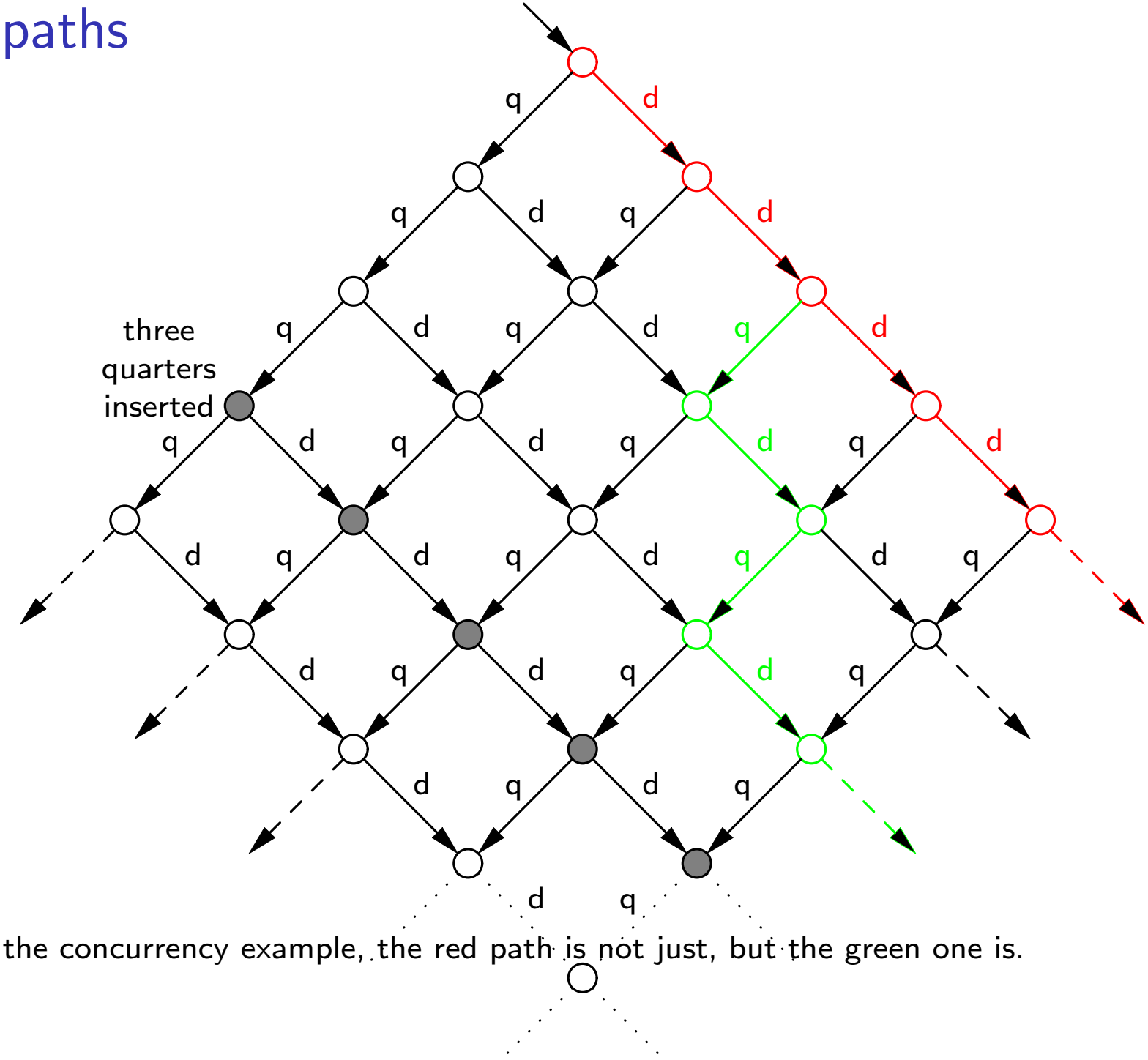
When assuming *fairness* it holds for both examples.

Just paths



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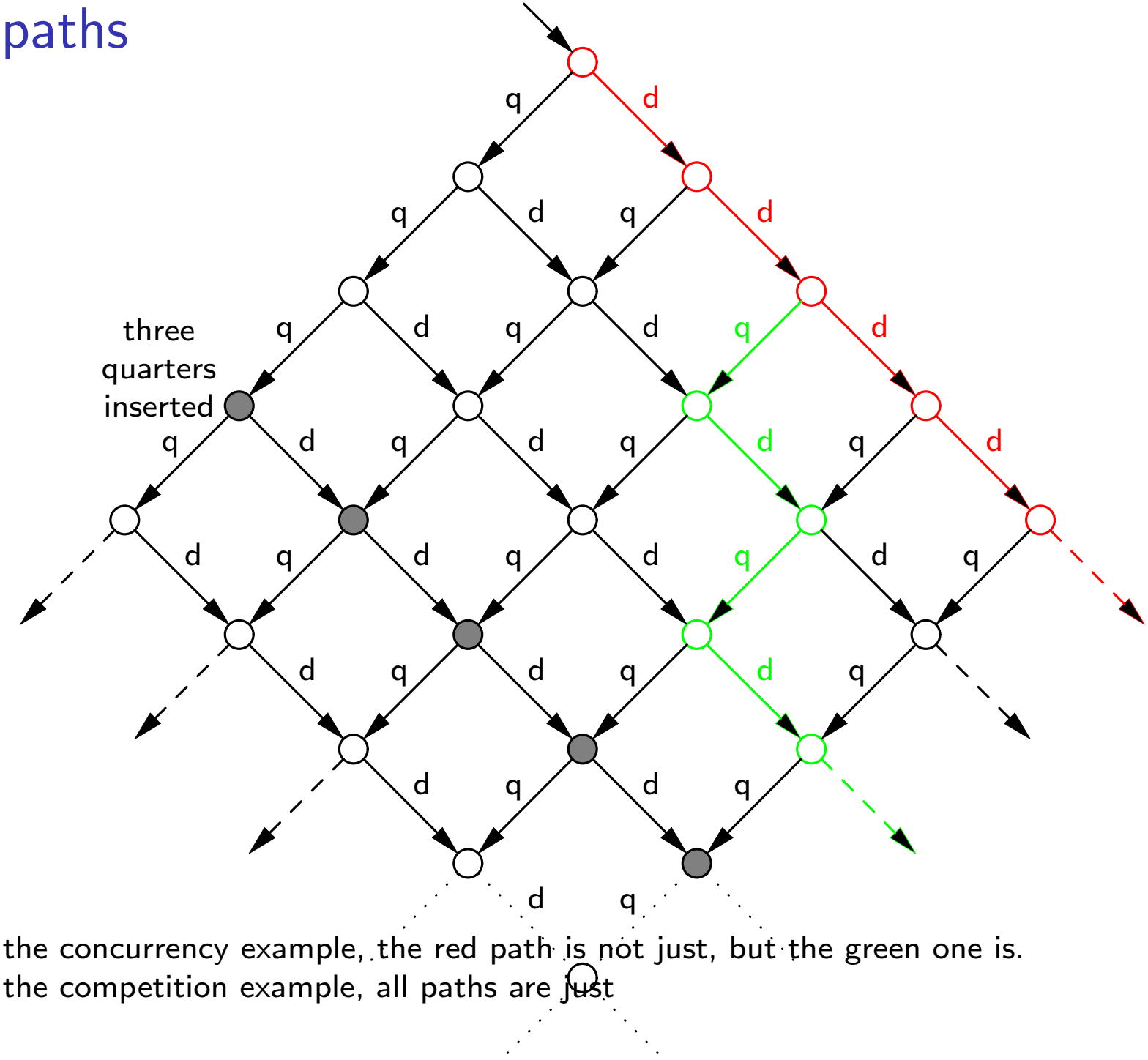

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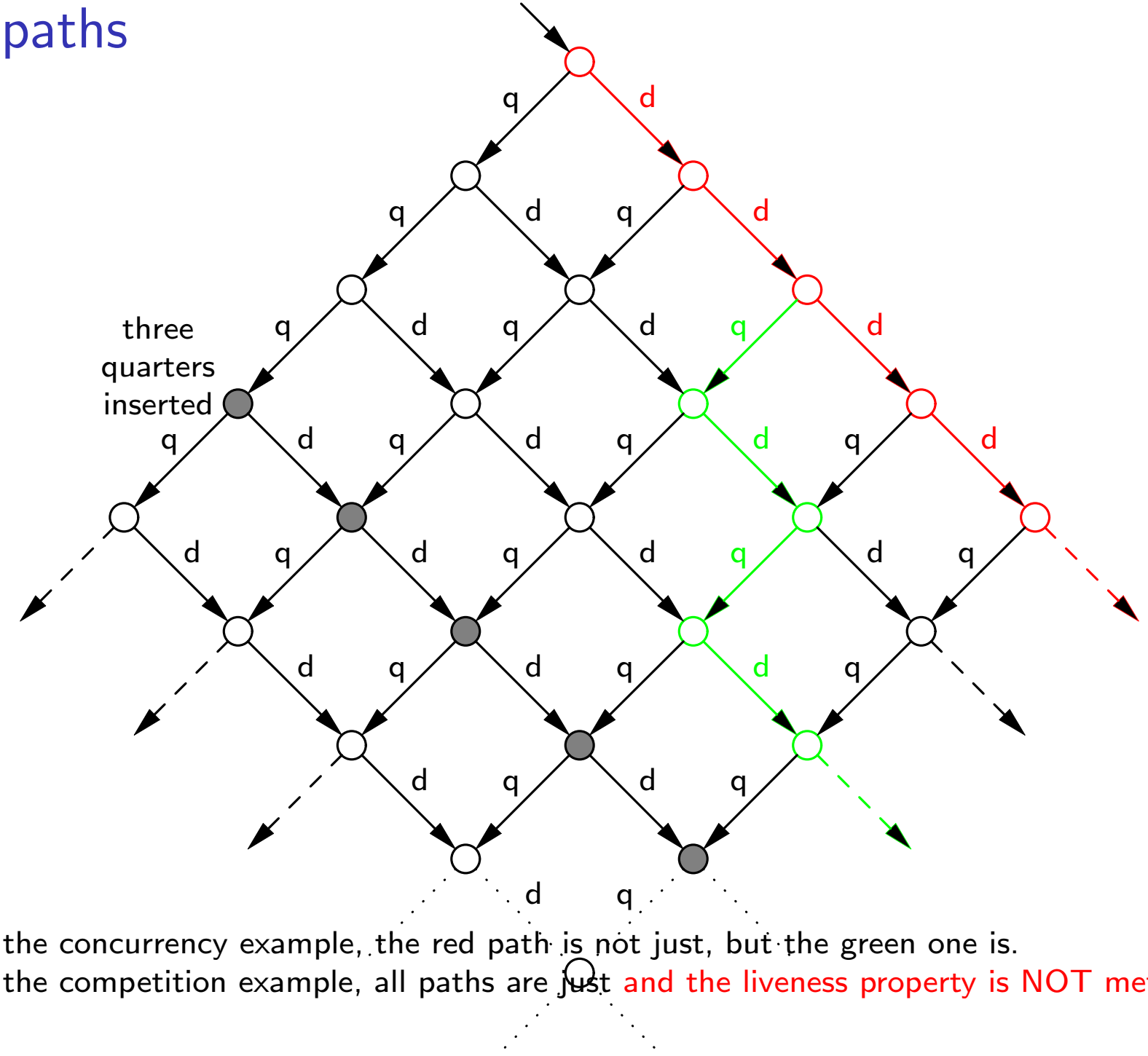

In the concurrency example, the red path is not just, but the green one is.
In the competition example, all paths are just and the liveness property is NOT met.

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Much contemporary research fails to distinguish justness and fairness.
This can lead to unwarranted conclusions and system failures.

Research agenda

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More importantly, we need new technologies to perform efficient verifications in this revised setting.

- Model checking (based on temporal logic)
- Induction principles $\leftarrow$ requires new ideas
- Syntactic formats to ensure compositionality

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  A complex system is verified, by verifying its parts, and composing the verified parts in a black-box manner.

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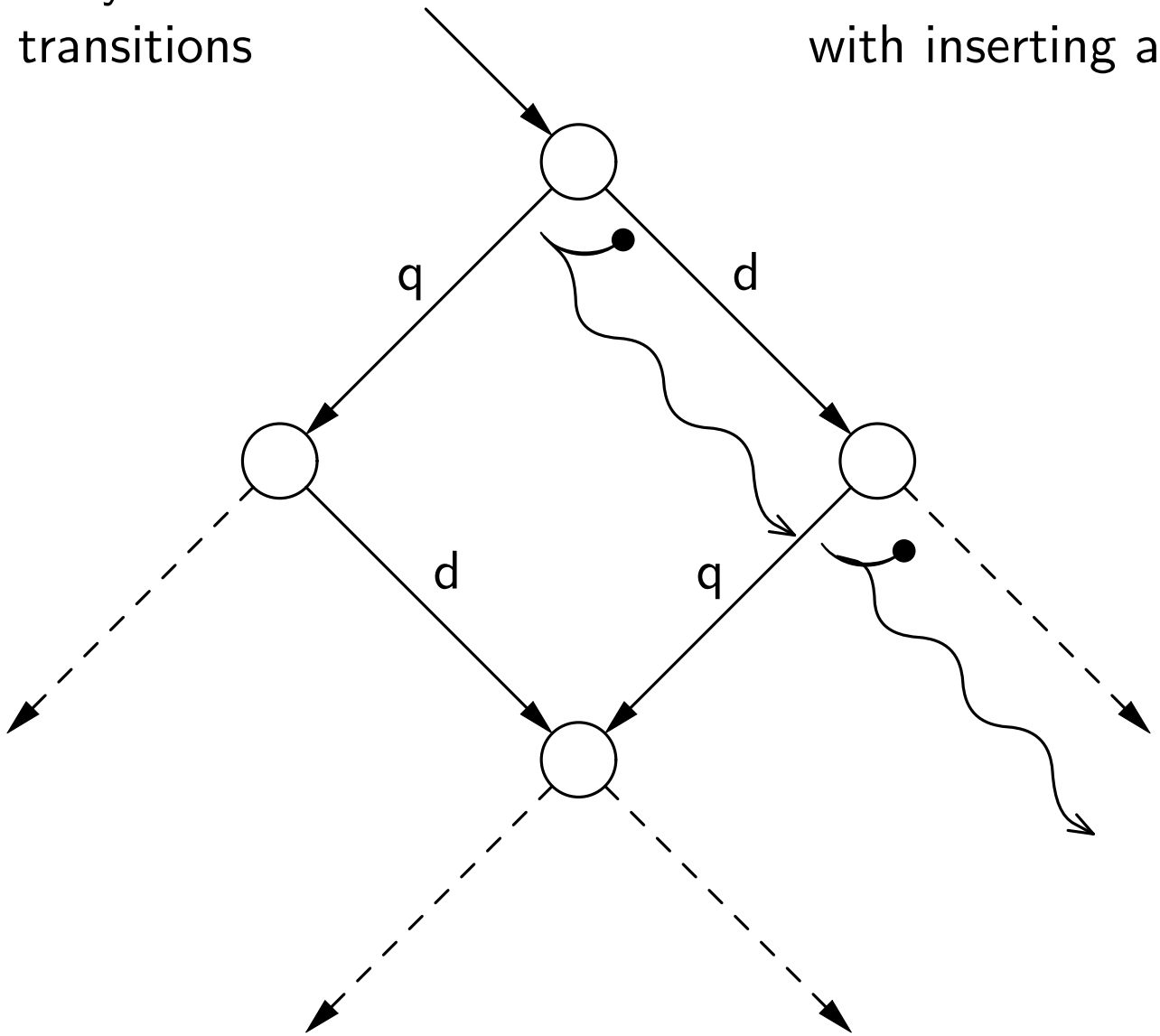
  Next to abstraction from internal activity, compositionality is the most powerful tool to attack the state-space explosion.
  A complex system is verified, by verifying its parts, and composing the verified parts in a black-box manner.
  Syntactic checks on code are known that guarantee that forms of compositional reasoning are warranted. But such work needs to be redone when factoring in justness.

Transition systems with successors

Transition systems
plus a ternary relation
between transitions

inserting a dime
does not interfere
with inserting a quarter



Formalising Justness

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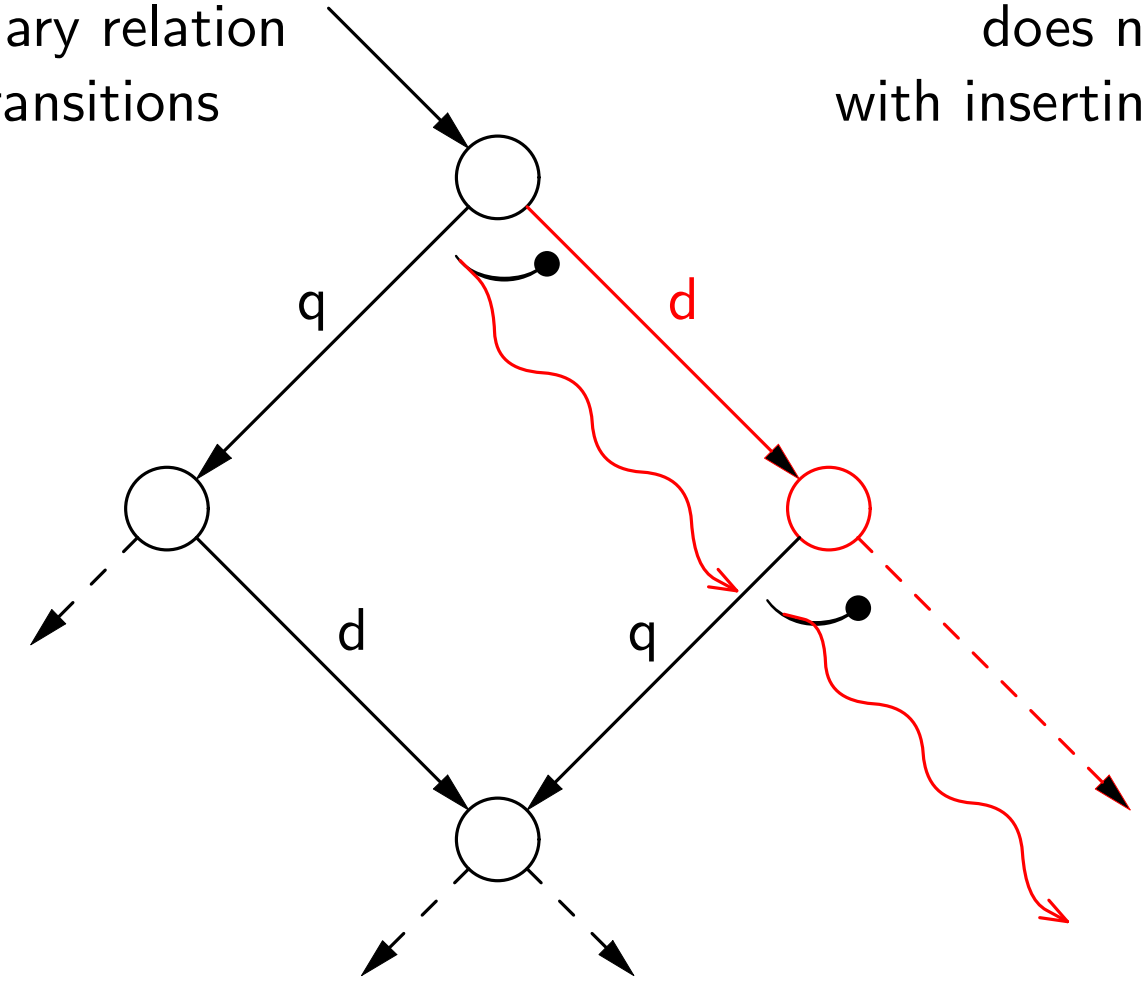

Justness: The system never follows a $\rightarrow$-path
that induces an infinite $\rightsquigarrow$-sequence.

Application: Verification of Mutual Exclusion Protocols

Mutual exclusion:
N parallel *threads* (or *processes* or *computers*)
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The code of the treads has *critical sections*.
A mutual exclusion protocol aims to assure that:

- *mutual exclusion*: at any given time, at most one thread will be in its critical section.
- *deadlock freedom*: Whenever at least one thread wants to enter its critical section, eventually some thread will enter its critical section.
- *starvation freedom*: Whenever a thread wants to enter its critical section, eventually it will enter its critical section.

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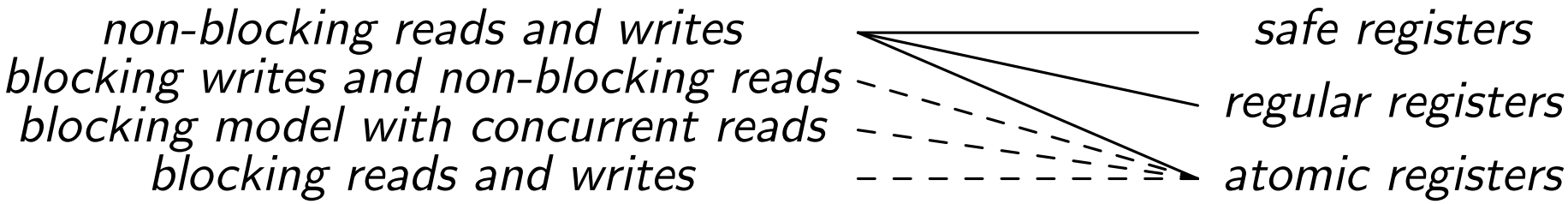

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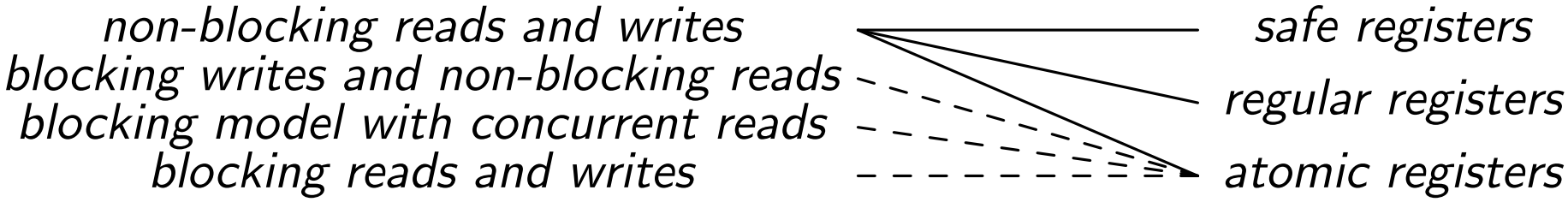

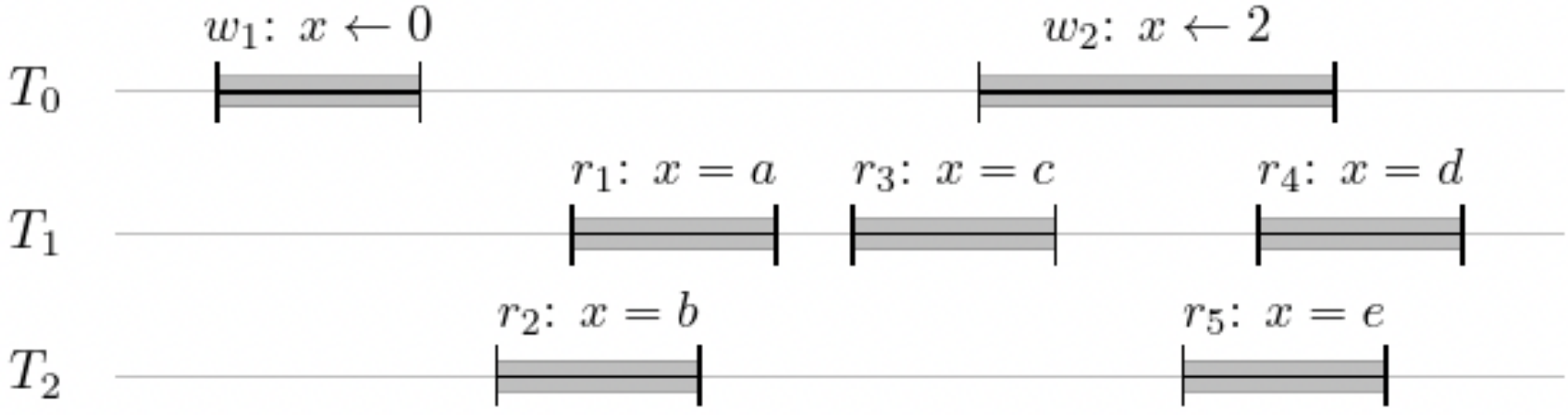

Model checking

Traditional verification of mutual exclusion algorithms:
pen-and-paper proofs using behavioural reasoning.

> *"the behavioral reasoning used in our correctness proofs, and in most other published correctness proofs of concurrent algorithms, is inherently unreliable" [Lamport 1986]*

This is especially the case when dealing with non-atomic registers.

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Alternatives: interactive theorem proving or model checking.

Model checking: Precise modelling requires great care,
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But only finite state spaces.
So no bakery protocol
and we can check only for a small number of threads.

Results

Algorithm	*# threads*	*Safe*	*Regular*	*Atomic*			
		T	*T*	*T*	*S*	*I*	*A*
Anderson	2	S	S	S	S	M	M
Aravind (BLRU)	3	S	S	S	M	M	M
Aravind (BLRU, alt.)	3	S	S	S	S	M	M
Attiya-Welch (orig.)	2	D	S	S	D	M	M
Attiya-Welch (orig., alt.)	2	S	S	S	D	M	M
Attiya-Welch (var.)	2	M	M	S	D	M	M
Attiya-Welch (var., alt.)	2	S	S	S	D	M	M
Burns-Lynch	3	D	D	D	D	M	M
Dekker	2	M	M	S	D	M	M
Dekker (alt.)	2	M	M	S	S	M	M
Dekker (RW-safe)	2	S	S	S	D	M	M
Dijkstra	3	M		D	M	M	M

Results (continued)

Kessels	2	X	X	S	S	M	M
Knuth	3	M	S	S	M	M	M
Lamport 1-bit	3	D	D	D	D	M	M
Lamport 3-bit	3	S	S	S	S	M	M
Peterson	2	X	X	S	S	M	M
Peterson (new sol., int.)	3	D					
Peterson (new sol., bit)	3						
Szymanski (flag)	3	X	X	S	S	M	M
Szymanski (flag bit)	3	X	X		X		
Szymanski (3-bit lin. wait)	3	X	X		X		
Szymanski (3-bit lin. wait, alt.)	2	S	S	S	S	M	M
Szymanski (4-bit robust)	3	X	X		X		
Szymanski (4-bit robust reset)	3	X	X				